MTH 151, Exam #1, Fall 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Let A be the set of letters in the name CAROLINGINIAN and let B be set of letters in the name PERPENDICULAR.
   a. List the elements in set A using proper set notation. (3 points)

$A = \{C, A, R, O, L, I, N, G\}$

   b. List the elements in set B using proper set notation. (3 points)

$B = \{P, E, R, N, D, I, C, U, L, A\}$

   c. Find $A \cap B$. (3 points)

$\{A, C, I, L, N, R\}$

   d. Find $A \cup B$. (3 points)

$\{A, C, D, E, G, I, L, N, O, P, R, U\}$

   e. What is the cardinality of set A, i.e. $n(A) = |A|$? (2 points)

8

   f. What is the cardinality of set $A \cup B$? (2 points)

12

   g. What is the cardinality of $A \times B$? (Do not attempt to list all the elements, just say how big the set is.) (3 points)

$8 \times 10 = 80$

2. Answer the following questions about sets:

a. List the elements in the set $C = \{x | x \text{ is an even counting number less than } 10\}$. (3 points)

$\{2, 4, 6, 8\}$

b. List the elements in set $D = \{x | x \text{ is an integer between } -1 \text{ and } 1 \text{ inclusive}\}$. (3 points)

$\{-1, 0, 1\}$

c. List the elements in $C \times D$. (4 points)

$\{(2,-1), (2,0), (2,1), (4,-1), (4,0), (4,-1), (6,-1), (6,0), (6,1), (8,-1), (8,0), (8,1)\}$

d. How many elements are in $C \times C \times C = C^3$? (3 points)

$4^3 = 64$

e. For each of the following questions, answer TRUE or FALSE. (1 point each)

i. $4 \in C$ true

ii. $C \cap D = \emptyset$ true

iii. $\emptyset \in C$ false

iv. $1 \subset D$ false

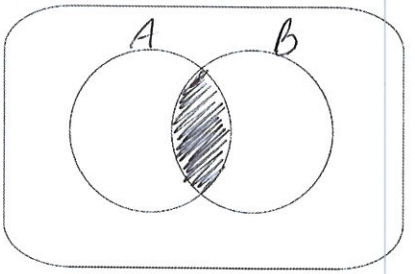
v. $\{(2,1), (2,0), (2,-1)\} \subset C \times D$ true

f. List all the subsets of D. [Hint: there are 8 of them.] (6 points)

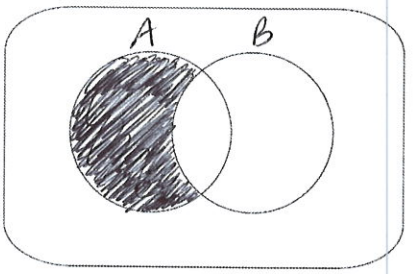
$\{-1, 0, 1\}$, $\{-1, 0\}$ $\{-1, 1\}$ $\{0, 1\}$,
$\{-1\}$, $\{0\}$, $\{1\}$, $\emptyset$

3. Draw a Venn Diagram that illustrates each of the following sets. (3 points each)

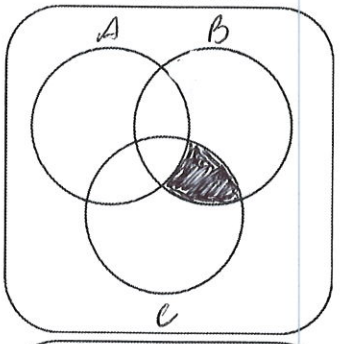
a. $A \cap B$



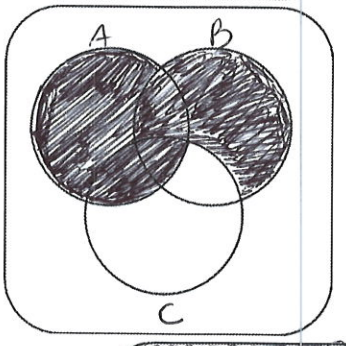
b. $A - B$



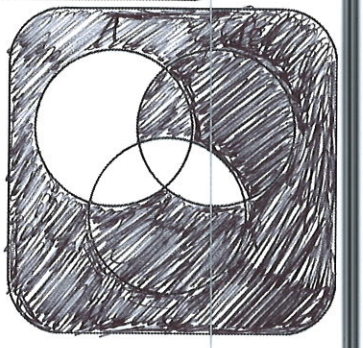
c. $(A' \cap B) \cap C$



d. $(B - C) \cup A$

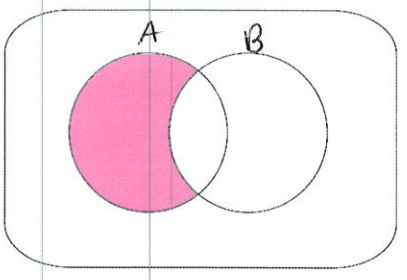


e. $(A' \cup B) - (C \cap B)$



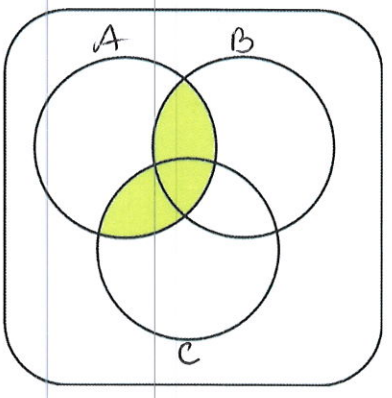
4. For each of the following Venn diagrams, write set notation that describes the indicated set. (3 points each)

a.



$A - B$

b.



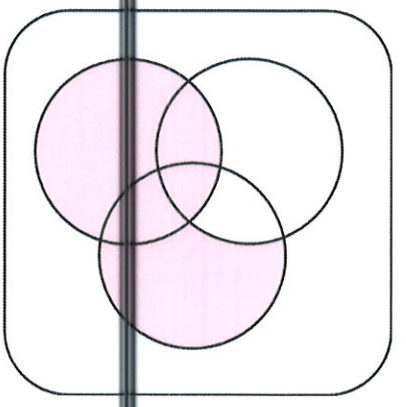
$(A \cap B) \cup (A \cap C)$

c.

$(A \cup B)'$

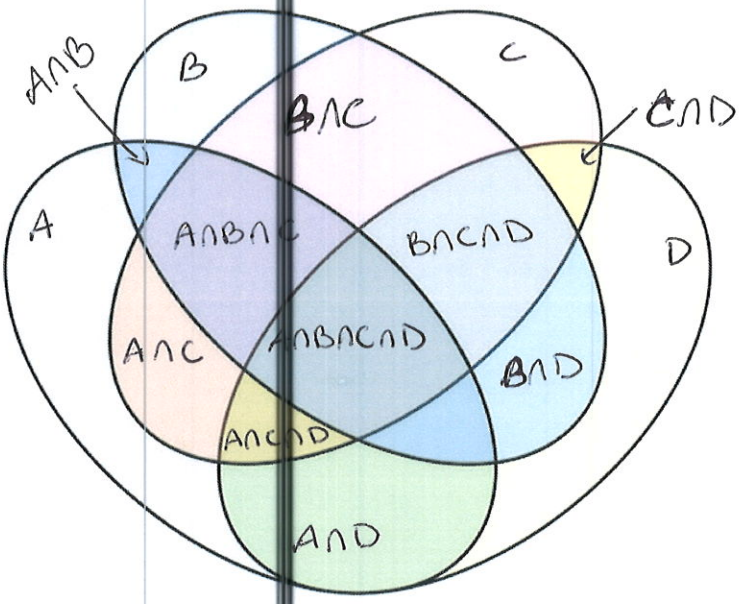
A B

d.



$A \cup (C - B)$

5. A 4-set Venn diagram is shown here. Label all the sections of the diagram with appropriate intersection notation. Use arrows if the region is too small to write in. Label the main 4 sets A, B, C, D. (6 points)



6. A survey was conducted among 75 patients admitted to a hospital cardiac unit during a two-week period. Let B be the set of patients with high blood pressure, C be the set of patients with high cholesterol levels, and S the set of patients that smoke cigarettes. Fill in the Venn diagram below using the following data, and then use the diagram to answer the questions that follow. (16 points)
   - The number of patients with high blood pressure was 47
   - The number of patients with high cholesterol was 46
   - The number of patients who smoke is 52.
   - The number of patients who smoke and have high blood pressure is 33
   - The number of patients who both have high blood pressure and high cholesterol is 31
   - The number of patients who have all three conditions is 21
   - The number of patients with exactly two conditions is 51

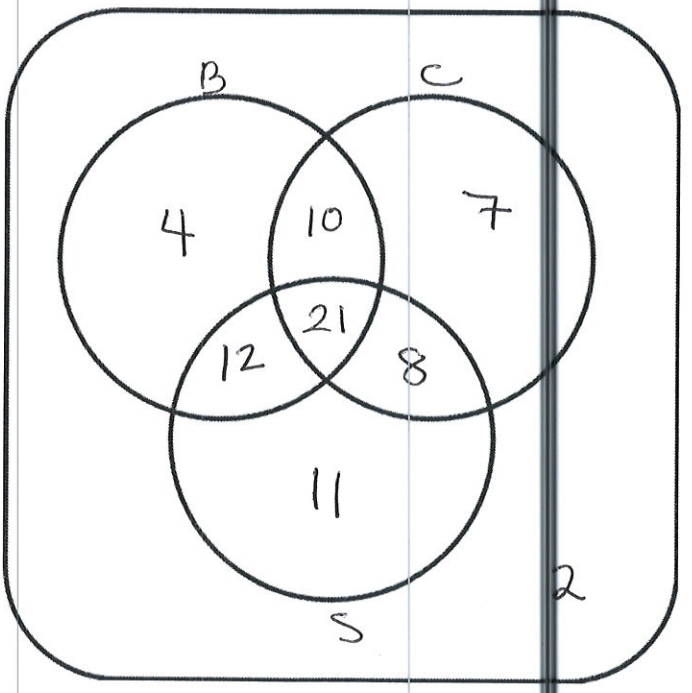
a. Find the number of patients who had either high blood pressure or high cholesterol, but not both.

$4 + 7 = 11$

b. Find the number of patients who had one or none of these conditions.

$2 + 11 + 4 + 7 = 24$

c. Find the number of patients who have none of these conditions. 2



7. Let p be the statement "She has green eyes," and let q be the statement "Andrew is 91 years old," and r be the statement "The cat is lonely." Use this information to translate the following symbolic statements into English sentences. (3 points each)

a. $p \wedge q$

She has green eyes and Andrew is 91 years old.

b. $\sim p \rightarrow q$

if She does not have green eyes, then Andrew is 91 years old.

c. $(p \vee \sim q) \leftrightarrow r$

She has green eyes or Andrew is not 91 years old if and only if the cat is lonely.

8. If $a(x)$ is the statement "x has apples", and $b(x)$ is the statement "x has blueberries", and $c(x, y)$ is the statement "x contains y", then translate the following symbolic statements into English. (3 points each)

a. $(\forall q)(a(q))$

Every q has apples

b. $(\exists y)(\sim b(y))$

Some y does not have blueberries

c. $\sim(\forall x)(\exists y)(c(y, x))$

it is not the case that every x contains some y.

9. Construct truth tables for each of the following statements.

a. $p \wedge \sim q$ (4 points)

P	q	~q	P ∧ ~q
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

b. $(p \to q) \vee \sim r$ (6 points)

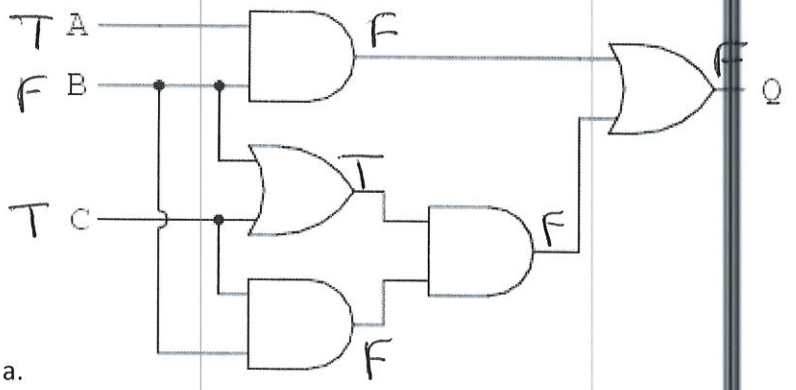
p	q	r	$p \to q$	$\sim r$	$(p \to q) \vee \sim r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

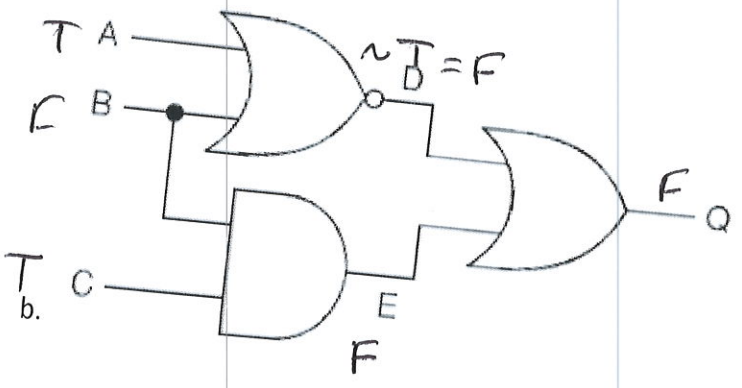
10. Explain in your own words the difference between "inclusive or" and "exclusive or". (3 points)

Inclusive or is like and/or in English, at least one is true, and both can be true. Exclusive or lets only one statement be true, not both

11. Find the truth value of the logic gates below using the fact that A is True, B is False, and C is True. (3 points each)

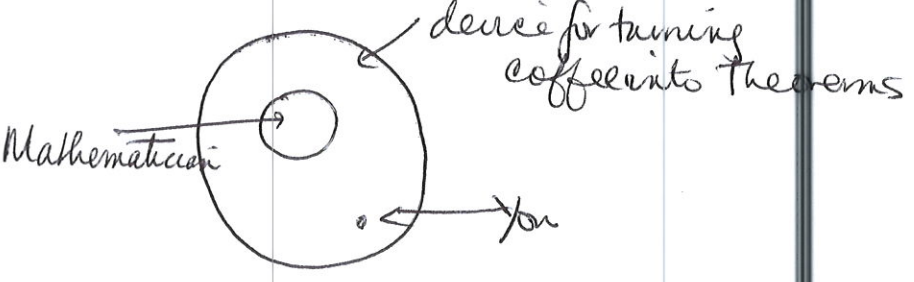
a.





12. Use either a Truth Table or an Euler diagram to determine the validity of the following argument. (8 points)

A mathematician is a device for turning coffee into theorems.
<u>You turn coffee into theorems.</u>
You are a mathematician.



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