MTH 151, Final Exam, Fall 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Let A be the set of letters in the name MATHEMATICAL and let B be set of letters in the name ENLIGHTENMENT. (5 points each)
   a. List the elements in set A using proper set notation.

$A = \{M, A, T, H, E, I, C, L\}$

   b. List the elements in set B using proper set notation.

$B = \{E, N, L, I, G, H, T, M\}$

   c. Find $A \cap B$.

$\{E, L, I, H, T, M\}$

   d. Find $A \cup B$.

$\{M, A, T, H, E, I, C, L, N, G\}$

   e. What is the cardinality of set A, i.e. $n(A) = |A|$?

8

   f. What is the cardinality of set $A \cap B$?

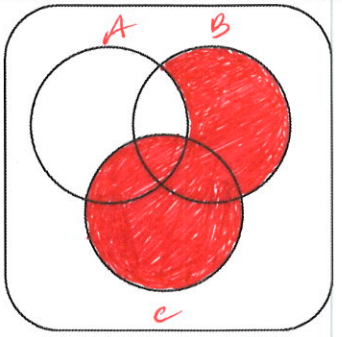
6

g. What is the cardinality of $A \times B$? (Do not attempt to list all the elements, just say how big the set is.)

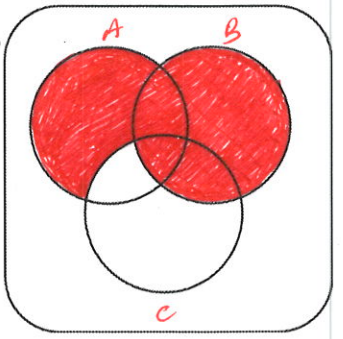
$8 \times 8 = 64$

2. Draw a Venn Diagram that illustrates each of the following sets. (5 points each)

a. $(A' \cap B) \cup C$



b. $(A - C) \cup B$



3. At a southern university, half of the 48 mathematics majors were receiving federal financial aid: (20 points)
   - 5 had Pell grants
   - 14 participated in College Work Study programs
   - 4 had TOPS scholarships
   - 2 had TOPS scholarships and participated in Work Study.
   - Those with Pell grants had no other financial aid.

a. How many of the 48 math majors had no federal aid?

27

b. How many had more than one of these three forms of aid?

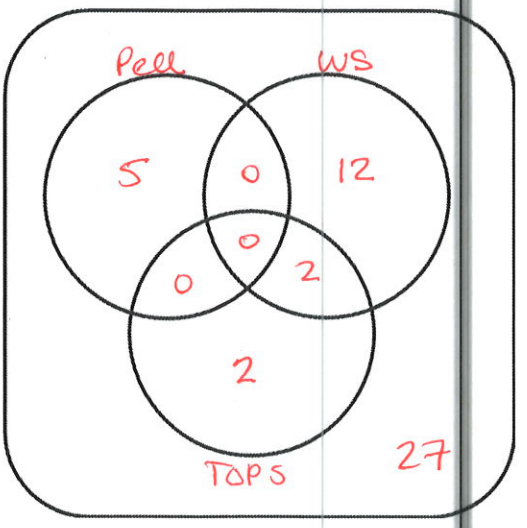
2

c. How many had a TOPS scholarship or Work Study?

16

d. How many had exactly one of these three forms of aid?

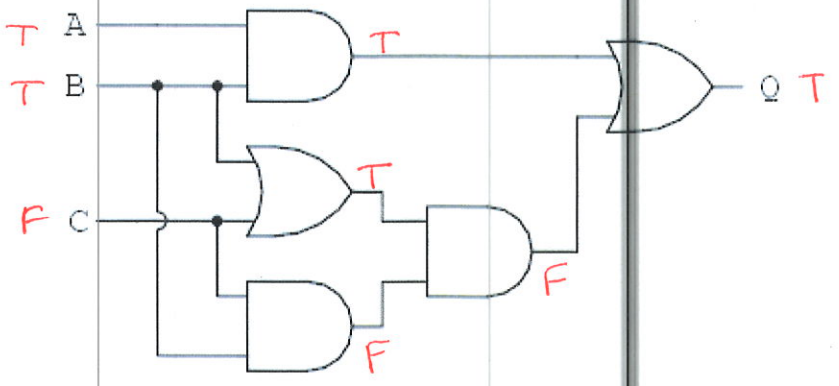
19



4. Construct truth tables for each of the following statements.
   a. $(p \rightarrow \sim r) \vee q$ (10 points)

p	q	r	~r	p → ~r	(p → ~r) ∨ q
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	F	T	F	T	T
F	F	F	T	T	T

5. Find the truth value of the logic gates below using the fact that A is True, B is True, and C is False. (8 points)



6. Write the following numerals in historical counting systems in the Hindu-Arabic system. (8 points each)

a.  23,527

b. $\overline{MCCLVII}DXCIV$ 1,257,594

c. 
9 × 20^4
9 × 20^3
2 × 20^2
4 × 20^1
8
= 1,512,888

7. Write the number 5,631 in the following numeral systems: (6 points each)

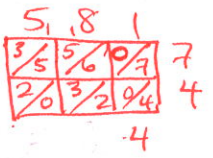
a. Babylonian

5631 − 3600 = 2031
2031 − 33 × 60 = 51

b. Chinese

五千六百三十一

8. Use the lattice method to calculate 581×74. (6 points)



42,994

9. Write the number 143 in the following bases: (6 points each)
   a. Base-2 (binary)

143 − 128 = 15
15 − 8 = 7

10001111

   b. Base-16 (Hexadecimal)

143/16 = 8 +
143 − 8×16 = 15

8F

10. Convert the number 6713_8 in the given base into base-10. (6 points)

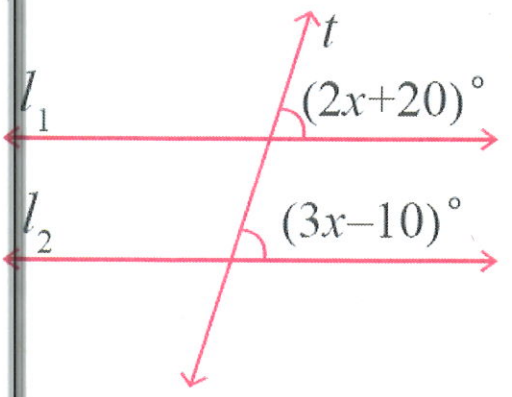
$6\times8^3 + 7\times8^2 + 1\times8 + 3$

$= 3531$

11. Use the shortened Binet formula $F_N = \left[\left[\left(\frac{1+\sqrt{5}}{2}\right)^N / \sqrt{5}\right]\right]$ to find F_{12}. (8 points)

144

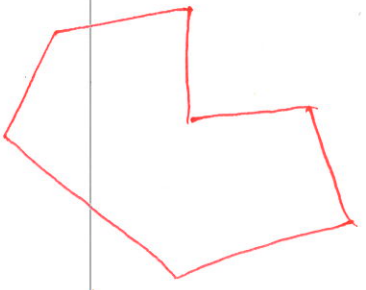
12. Find x if l_1 and l_2 are parallel. (10 points)



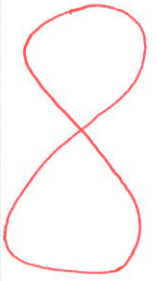
$2x+20 = 3x-10$
$-2x+10 \quad -2x+10$
$30 = x$

13. Give an example of a shape that is a polygon and one that is not. Explain why you classified each one the way you did. (10 points)

Simple closed



Polygon

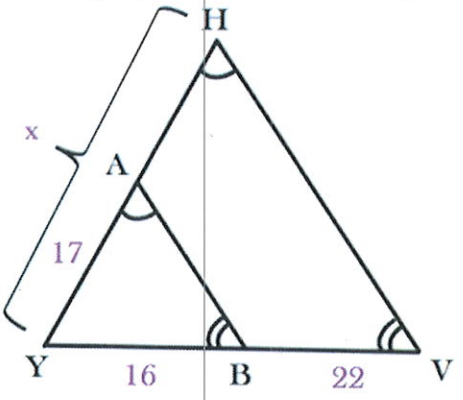


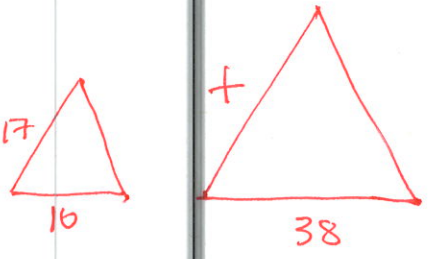
not straight lines
not simple

not a polygon

answers will vary

14. Use the properties of similar triangles to find the value of x. (12 points)



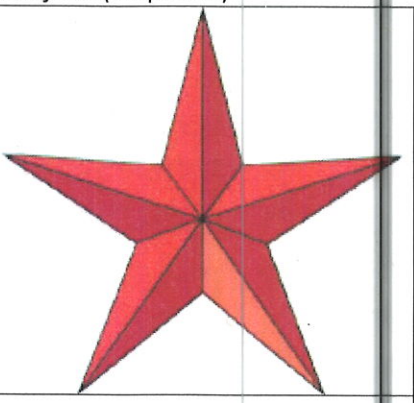
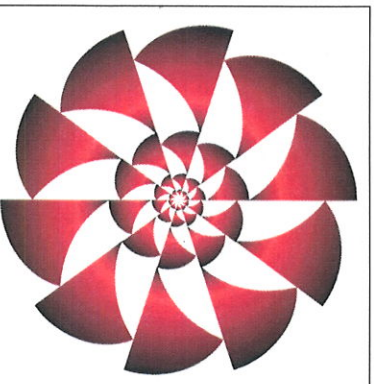


$\frac{x}{17} = \frac{38}{16}$

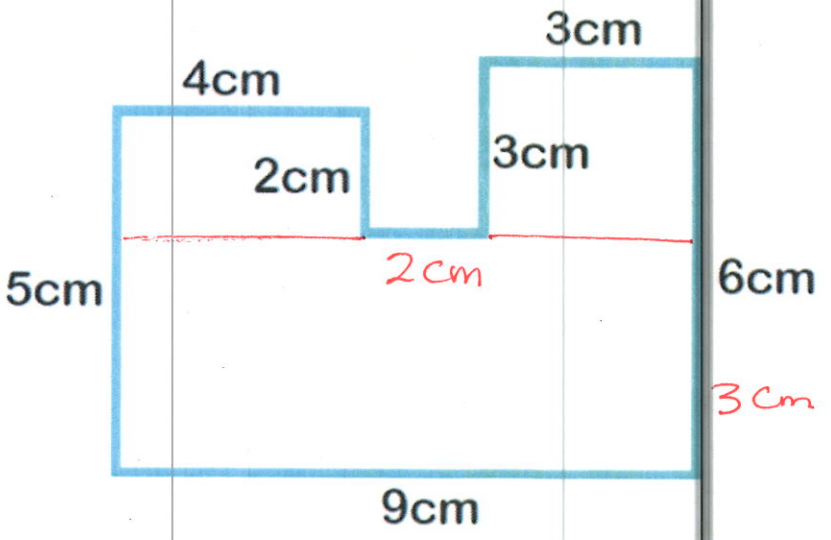
$16x = 646$

$x = 40.375$

15. Identify the symmetry of each object. (18 points)

		
Lines of Symmetry: 1	Lines of Symmetry: 5	Lines of Symmetry: 0
Rotational Symmetry: 1	Rotational Symmetry: 5	Rotational Symmetry: 10

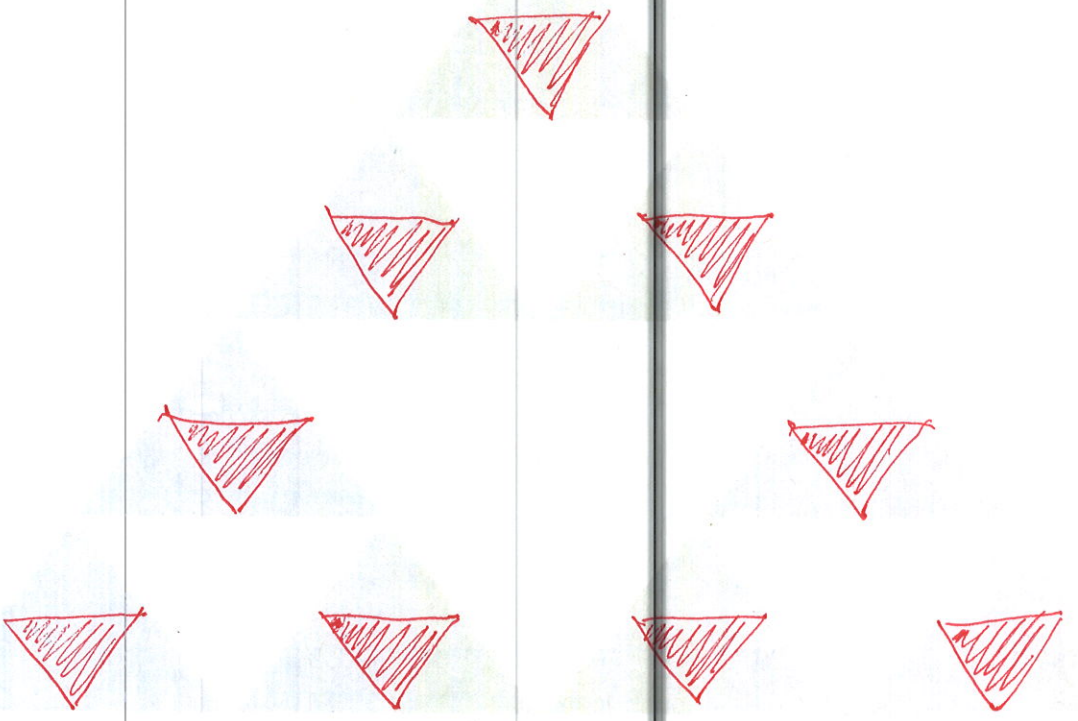
16. Find the area and perimeter of the following object. (10 points)



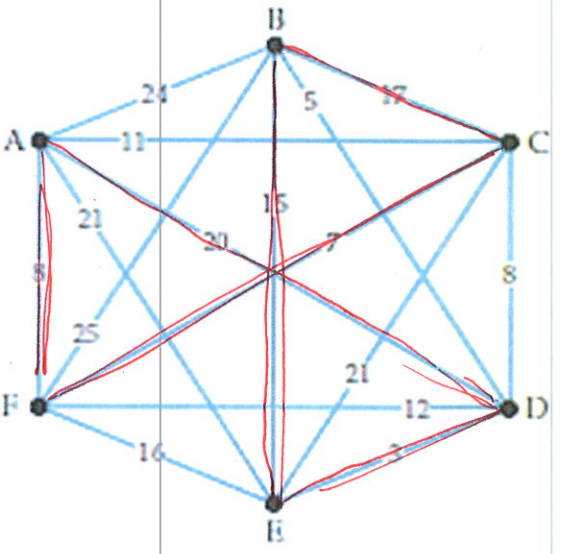
perimeter = 34 cm

area = $3\times3 + 4\times2 + 3\times9 = 44\ cm^2$

17. Draw in the next stage of the Sierpinski Gasket. (10 points)

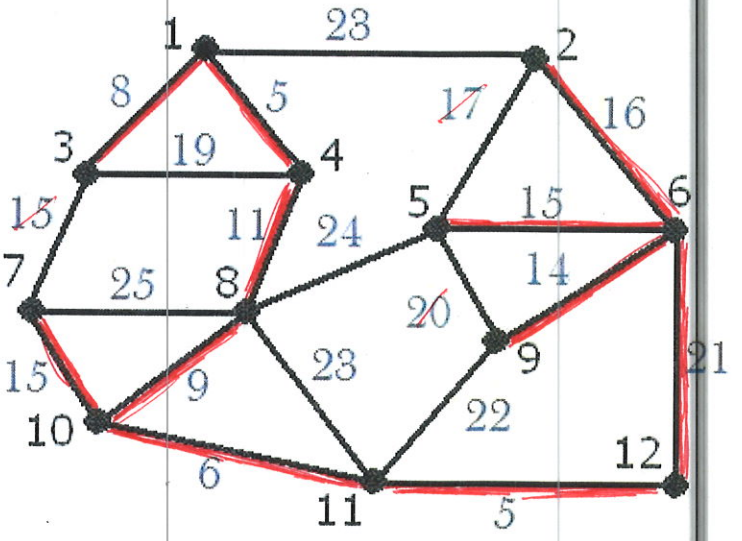


18. Use Nearest Neighbor, starting at point C, to estimate the least costly Hamilton Circuit. Be sure to state the final cost of the circuit. (10 points)



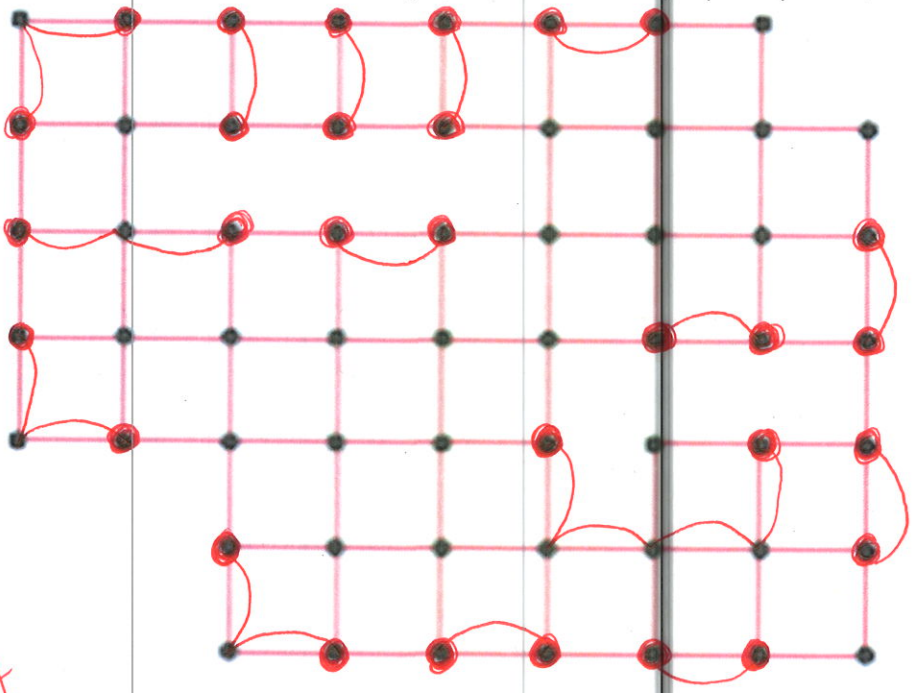
C F A D E B C
7+8+20+3+15+17
= 70

19. Use Kruskal's algorithm to find the minimum spanning tree. Be sure to state the cost of the minimum spanning tree. (10 points)



5+5+6+8+9+11+14+15+15+16+21 = 125

20. Label all odd vertices, then Eulerize the graph shown below. (10 points)



answers will vary

21. Find the interest earned on $6000 deposited into an account earning 2% annual interest for 4 years, compounded: (7 points each)

a. quarterly

$6000\left(1+\frac{.02}{4}\right)^{16} = \$6498.43$

or

$N = 16$
$I = 2$
$PV = 6000$
$PMT = 0$
$FV = 6498.43$
$P/Y = C/Y = 4$

b. continuously

$6000e^{.02(4)} = \$6499.72$

22. Find the monthly payment on a mortgage of $300,000 paying interest of 4.5% annual interest compounded monthly. (10 points)

for 30 years

$N = 12 \times 30 = 360$
$I = 4.5$
$PV = 300000$
$PMT = 1520.06$
$PV = 0$
$P/Y = C/Y = 12$

$\$1520.06$

$I = Prt$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$payoff = (k+1)R - u$

$R = \frac{P\left(\frac{r}{n}\right)}{1-\left(\frac{n}{n+r}\right)^{nt}}$

$V = \frac{(1-t)R[(1+r)^n-1]}{r}$

$r_{eff} = \left(1+\frac{r}{n}\right)^n - 1$

$A = Pe^{rt}$

$u = \frac{k(k+1)}{n(n+1)}F$

$NAV = \frac{A-L}{N}$

$V = \frac{R\left[(1+r(1-t))^n-1\right]}{r}$

$A = P(1+rt)$

$u = kR\left(\frac{h}{100+h}\right)$

$h = \frac{nt\left(\frac{r}{n}\right)100}{1-\left(1+\frac{r}{n}\right)^{-nt}} - 100$

$V = R\left[\frac{(1+r)^n-(1+i)^n}{r-i}\right]$