MTH 151 Homework #2 Key

1a. I did not buy a lottery ticket

b. If I buy a lottery ticket, I will win the million-dollar jackpot.

c. I buy a lottery ticket if and only if I won the million-dollar jackpot.

d. I did not buy a lottery ticket and I did not win the million-dollar jackpot.

e. I bought a lottery ticket or I won the million-dollar jackpot.

f. I bought a lottery ticket and I won the million dollar jackpot

g. If I did not buy a lottery ticket, then I did not win the million-dollar jackpot.

h. I did not buy a lottery ticket or I both bought a lottery ticket and I won the million-dollar prize.

2a. $\sim p$

b. $p \wedge \sim q$

c. $p \to q$

d. $\sim p \to \sim q$

e. $q \to p$

f. $q \wedge \sim p$

g. $q \to p$

3a. 2 b. $2^6 = 64$ c. 16 d. 16

4a.

p	~p
T	F
F	T

b.

p	q	$p \leftrightarrow \sim q$	$p \vee q$	$(p \leftrightarrow \sim q) \wedge (p \vee q)$
T	T	F	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	F

4c.

p	q	r	s	p∧q	r→s	(p∧q) ⊻ (r→s)
T	T	T	T	T	T	F
T	T	T	F	T	F	T
T	T	F	T	T	T	F
T	T	F	F	T	T	F
T	F	T	T	F	T	T
T	F	T	F	F	F	T
T	F	F	T	F	T	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	T	F	F	F	F
F	T	F	T	F	T	T
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	T	F	F	F	F
F	F	F	T	F	T	T
F	F	F	F	F	T	T

d.

p	q	(p∨q)	p ⊻ (p∨q)
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

e.

p	q	r	(p∨q)	∼r	(p∨q)∧∼r
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	F	F
T	F	F	T	T	T
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	F	T	F

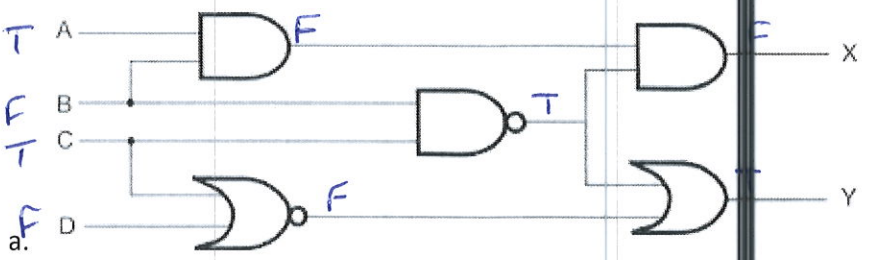
5a. if C is the knave, then A is the knight, and B the spy.

b. no solution

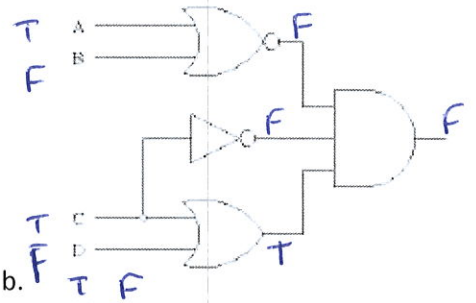
c. if A is knave, B could be spy and C the knight.

d. Any of them could be saying truth (knight) and other two lying.

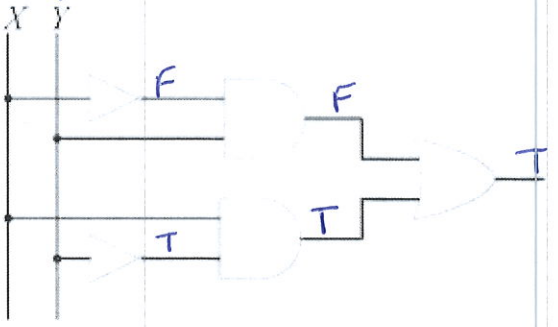
a.



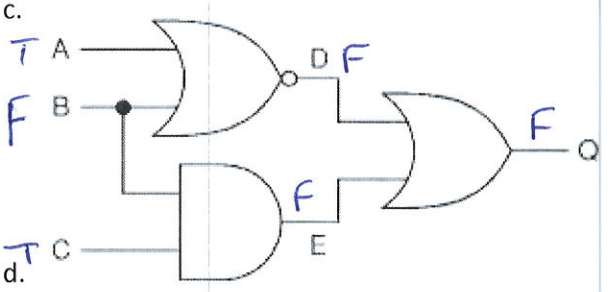
b.



c.



d.



7. Use De Morgan's Laws to find the negation of "James is young and strong."

8. Proofs.
   a. Show that the statement $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology.
   b. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.
   c. Show that the negation of a tautology is unsatisfiable. (You may do this with a specific tautology.)

9. Determine the truth value for each statement. If the statement is false, find a counterexample.
   a. $\forall n(n^2 \geq 0), n \in Z$
   b. $\exists n(n^2 = 2), n \in Z$
   c. $\exists !x(x + 3 = 2x), x \in R$
   d. $\exists x(x^2 = -1), x \in R$
   e. $\forall x(x^2 \neq x), x \in R$
   f. $\exists !x(x^2 = 1), x \in R$

6 on attached page.

7. James is young = p and James is strong = q

$\sim(p \wedge q) = \sim p \vee \sim q$

either James is not young or James is not strong

8. a.

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

← all true ∴ tautology

b.

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$	$(p \vee q)$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

these columns are the same

c. a tautology is always true so our truth table as in 8a is all T's

$\sim[(p \wedge q) \to (p \to q)]$

F
F
F
F

since negation flips all the truth values, thus it can never be true.

9. a. T b. false c. true d. false e. false f. false

10a. $(\forall x)$(x studies discrete mathematics)

b. $(\exists x)$(x is older than 21 years)

c. $(\forall x)(\exists y)$(x has the same name as y)

d. $(\forall x)$(there is no x who has taken a course in logic programming)

e. $(\forall x)$(x is not perfect)

f. $(\exists x)$(x is not in the correct place)

g. $(\exists x)$(x is not in the correct place $\wedge$ x is in excellent condition)

11a. Someone sent an email message to someone (else).

b. Everyone has sent an email to someone.

b. There is someone everyone sent a message to.

d. Everyone sent an email message to everyone (else).

12a. $(\forall x)(L(x, \text{Jerry}))$

b. $(\forall x)(\exists x)(L(x, y))$

c. $(\exists x)(\neg L(\text{Lydia}, x))$

d. $\neg(\exists x)(\forall y)(L(x, y))$

e. $(\exists x)[(\exists y)(y \neq x)](L(\text{Lynn}, x) \wedge$

f. $(\exists x)(\neg\forall y)(L(y, x) \rightarrow L(\text{Lynn}, y))$

13a. $1+1 = 1-1$ false

b. true $(y=0)$, x anything

c. true $n=1$

d. false

e. $1-y = 1-y$

$y = -y$ false

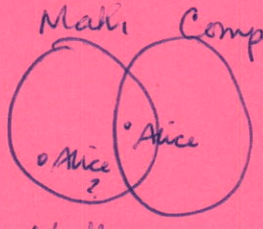
f. true

g. true

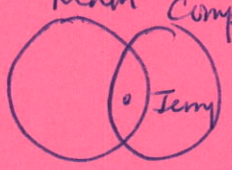
14a. $\exists y \exists x (\sim P(x,y) \wedge \sim Q(x,y))$

b. $\exists x (\forall y \exists z \sim P(x,y,z) \vee \forall z \exists y \sim P(x,y,z))$

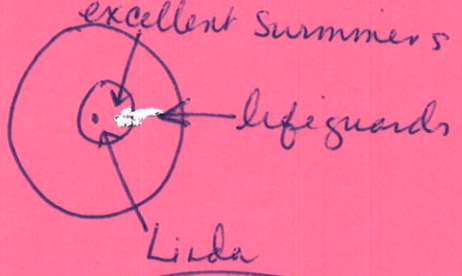
15. a.



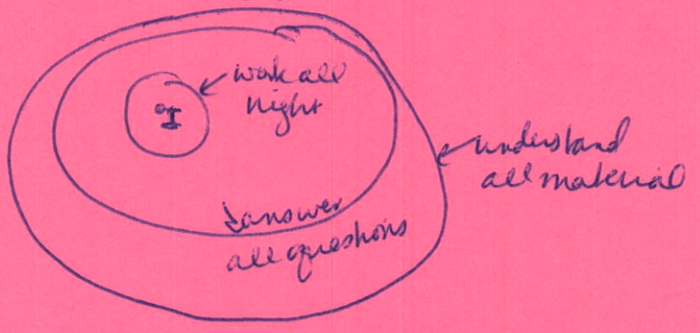
b.



c.



d.



16. a. It rained on Thursday

b. I am clever

e. rabbits are not rodents
mice gnaw their food

d. nothing

17. There is someone who is happy, but it need not be Lola

18a. converse not valid

b. converse not valid

c. not valid converse

19. Simplification false here (line iii)
the error is repeated on line v.

$P(c) \vee Q(c)$ does not mean $P(c)$ is true