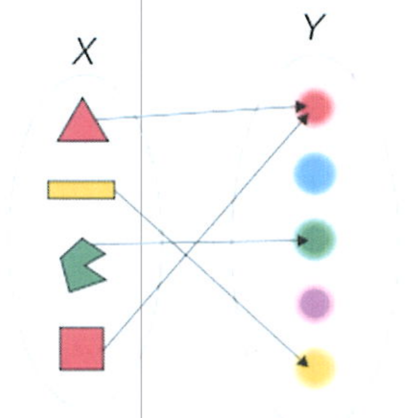
MTH 166, Exam #1, Fall 2017 Name KEY

**Instructions**: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing"; partial credit will not be possible.

1. For each of the following relations, determine i) the domain and range, ii) if the relation is a function, iii) if it is a function, is its inverse also a function. (2 points each)



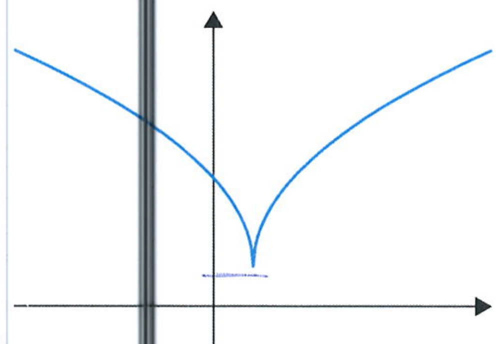

a.

function

D: {△, ▭, ⬠, □}

R: {red, green, yellow}

inverse not a function



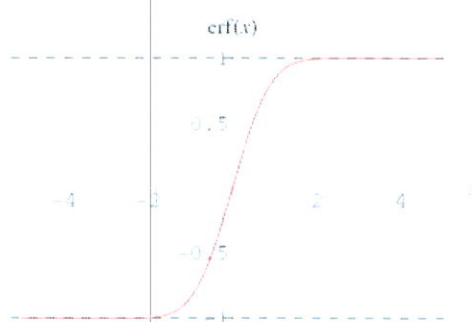
c.

function

D: all reals

R: [1, ∞)

inverse not a function



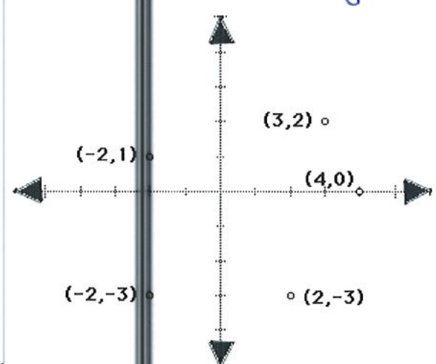

b.

function

D: $(-\infty, \infty)$

R: $(-1, 1)$

inverse is a function




d.

not a function

D: {-2, 3, 4, 2}

R: {1, -3, 2, 0}

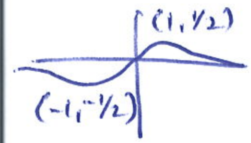
inverse is not a function

2. For each of the following functions, determine i) any intervals on which the function is increasing, ii) intervals on which the function is decreasing, iii) intervals on which the function is constant, iv) any relative extrema (relative maxima or minima), v) symmetry (even, odd or neither). [Hint: it's helpful to sketch the graph.] (3 points each)

   a. $f(x) = \frac{x}{x^2+1}$

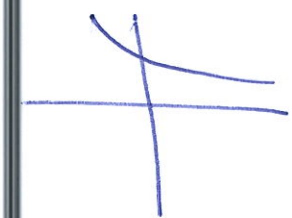
dec $(-\infty, 1) \cup (1, \infty)$
inc $(-1, 1)$      not constant

relative min $(-1, -\frac{1}{2})$
relative max $(1, \frac{1}{2})$
odd symmetry



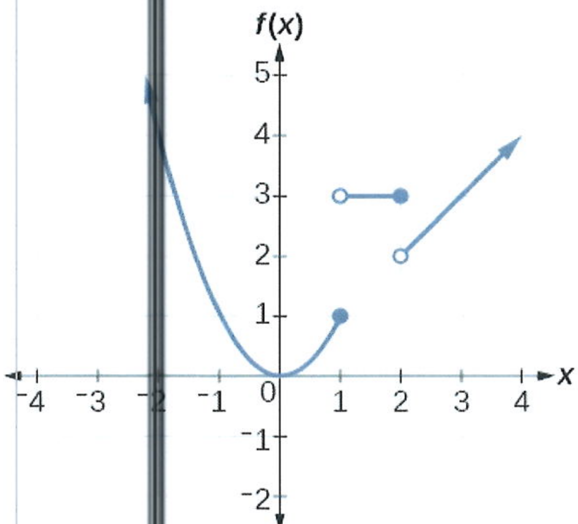

   b. $f(x) = |\sqrt{x+5} - 11|$

decreasing $(-5, \infty)$
not inc/constant
no relative min
relative max $(-5, 11)$
no symmetry



3. Write an equation of the piecewise graph shown. (4 points)

$$f(x) = \begin{cases} x^2 & x \le 1 \\ 3 & 1 < x \le 2 \\ x & 2 < x \text{ or } x > 2 \end{cases}$$



4. Find $\frac{f(x+h)-f(x)}{h}$ for $f(x) = -3x^2 + x - 1$. (5 points)

$$\frac{-3(x+h)^2 + (x+h) - 1 - (-3x^2+x-1)}{h}$$

$$\frac{-\cancel{3x^2} - 6xh - 3h^2 + \cancel{x} + h - \cancel{1} + \cancel{3x^2} - \cancel{x} + \cancel{1}}{h} = \frac{-6xh - 3h^2 + h}{h} =$$

$$\frac{\cancel{h}(-6x-3h+1)}{\cancel{h}} = -6x - 3h + 1$$

5. Find an equation of the line with the following properties: (3 points each)
   a. Passing through the points $(-2,-5)$ and $(6,-5)$.

$$\frac{-5-(-5)}{6-(-2)} = 0 \qquad \boxed{y = -5}$$

   b. Perpendicular to the line $3x + 4y = 12$ and passing through $(1,5)$.

$$y = -\frac{3}{4}x + 3 \qquad \perp \Rightarrow \frac{4}{3}$$

$$\boxed{y - 5 = \frac{4}{3}(x-1)}$$

   c. Parallel to $y = 7$ and passing through $(2,-3)$.

$$\boxed{y = -3}$$

6. If $f(x) = |x|$, write the function that has the following transformations applied: (3 points)
   a. Shift left 9 units $|x+9|$
   b. Reflect over the $x$-axis $-|x+9|$
   c. Compress by a factor of 3 $-\frac{1}{3}|x+9|$
   d. Shift down by 2

$$\boxed{g(x) = -\frac{1}{3}|x+9| - 2}$$

7. Shown is the function $f(x)$. Sketch the graph of $2f(-x+1)+3$. (5 points)

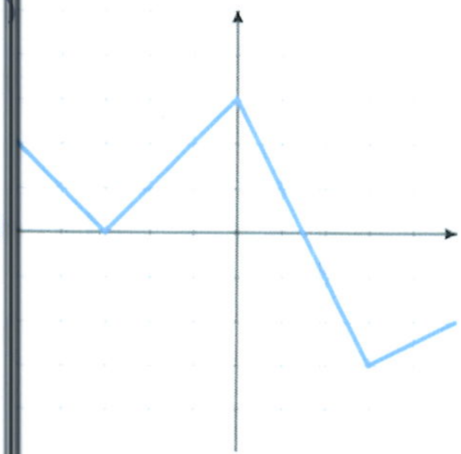
$(-5,2) \to (-4,2) \to (4,2) \to (4,4) \to (4,7)$ $\quad -(x-1)$

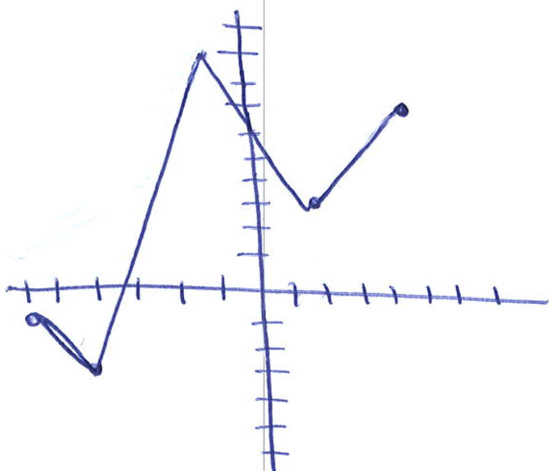
$(-3,0) \to (-2,0) \to (2,0) \to (2,0) \to (2,3)$

$(0,3) \to (1,3) \to (-1,3) \to (-1,6) \to (-1,9)$

$(3,-3) \to (4,-3) \to (-4,-3) \to (-4,-6) \to (-4,-3)$

$(5,-2) \to (6,-2) \to (-6,-2) \to (-6,-4) \to (-6,-1)$





8. Given $f(x)=x^2+1, g(x)=\sqrt{x-4}, h(x)=x+\frac{1}{x}$, find the following functions and state the domain (3 points each)

a. $(g+h)(x)$

$\sqrt{x-4}+x+\frac{1}{x}$

D: $x \geq 4$

b. $\left(\frac{f}{h}\right)(x)$

$$\frac{x^2+1}{x+\frac{1}{x}} \cdot \frac{x}{x} = \frac{x^3+x}{x^2+1} = \frac{x(\cancel{x^2+1})}{\cancel{x^2+1}} = x$$

D: $x \neq 0$

c. $(g \circ f)(x)$

$\sqrt{x^2+1-4} = \sqrt{x^2-3}$

$x^2-3 \geq 0$

$x^2 \geq 3$

D:

$x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$

9. Find the inverse of $f(x) = \frac{x+1}{x-2}$. Sketch the graph and its inverse on the same graph. Describe the symmetry you see. (5 points)

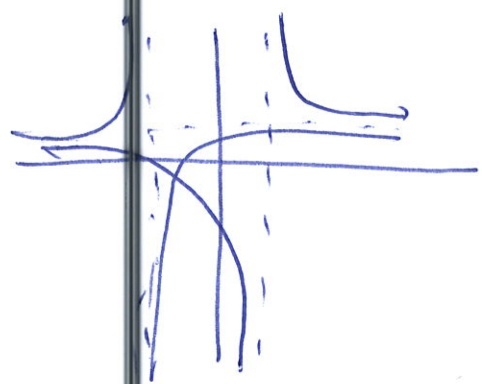
$x = \frac{y+1}{y-2}$

$xy - 2x = y+1$

$xy - y = 2x+1$

$y(x-1) = 2x+1$

$y = f^{-1}(x) = \frac{2x+1}{x-1}$



Symmetry across line $y=x$

10. The endpoints of a circle's diameter are $(-3, -4)$ and $(6, -8)$. Find the center of the circle, its radius, and equation in standard form. (4 points)

midpoint = center = $\left(\frac{-3+6}{2}, \frac{-4-8}{2}\right)$

$= \left(\frac{3}{2}, -6\right)$

radius = d = $\sqrt{\left(\frac{3}{2}+3\right)^2 + (-6+8)^2} = \sqrt{\left(\frac{9}{2}\right)^2 + 4} = \sqrt{\frac{81}{4}+4}$

$= \sqrt{\frac{97}{4}} = \boxed{\frac{\sqrt{97}}{2}}$

$$\boxed{\left(x-\frac{3}{2}\right)^2 + (y+6)^2 = \frac{97}{4}}$$

11. Let $P(x, y)$ be a point on the graph of $y = x^2 - 8$. Express the distance $d$ from $P$ to the point $(2,4)$, as a function of the point's $x$-coordinate. Find the minimum distance graphically. (4 points)

$$d = \sqrt{(2-x)^2 + (4-(x^2-8))^2} =$$

$$\sqrt{4-4x+x^2+(12-x^2)} =$$

$$\sqrt{x^4-24x^2+144+4-4x+x^2} =$$

$$\sqrt{x^4-23x^2-4x+148}$$

relative min $(-3.35, 5.41)$
$(3.43, 1.45)$

relative max $(-.09, 12.17)$

12. Simplify the following expressions. Write each in standard form. (2 points each)

a. $7 - (-9 + 2i) - (-17 - i)$

$7+9-2i+17+i = 33-i$

b. $(5 - 2i)^2$

$25-20i+4i^2$

$21-20i$

c. $\frac{2+3i}{2+i} \cdot \frac{2-i}{2-i} = \frac{4+6i-2i-3i^2}{5} = \frac{7+4i}{5}$

$= \frac{7}{5} + \frac{4}{5}i$

13. Find the vertex, the $y$-intercept and any zeros (real and complex) of the function $f(x) = -2(x+1)^2 - 5$. Use that information to sketch the graph. [If the zeros are complex, you may need to plot additional points by hand; if so, use the symmetry of the graph.] (4 points)

vertex $(-1,-5)$

y-int $-2(1)^2-5=-7$

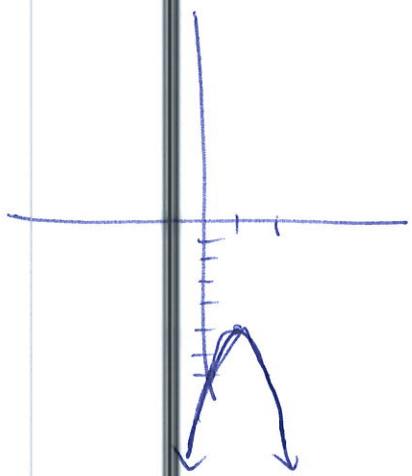
$-2(x^2+2x+1)-5$

$-2x^2-4x-2-5$

$0=-2x^2-4x-7$

$x = \dfrac{4 \pm \sqrt{16-4(-2)(-7)}}{-4}$

$= \dfrac{4 \pm \sqrt{-40}}{-4}$



no intercepts (x)

axis of symmetry $x=1$

14. Find all the possible rational zeros of the polynomial $f(x) = -x^3 - x^2 + 5x - 3$. Use them to factor the polynomial, and find all the real (and complex, if any) zeros. Write the polynomial in factored form. (6 points)

$\pm 1, \pm 3$

$$\begin{array}{r|rrrr} 1 & -1 & -1 & 5 & -3 \\ & & -1 & -2 & 3 \\ \hline & -1 & -2 & 3 & \boxed{0} \end{array}$$

$-(x-1)(x^2+2x-3)$

$-(x-1)(x+3)(x-1) = -(x-1)^2(x+3)$

15. Divide. Write the solution as $Quotient + \frac{Remainder}{Divisor}$. (4 points each)

a. $\frac{2x^5-8x^4+2x^3+x^2}{2x^3+1}$

$$\begin{array}{r} x^2-4x+1 \\ 2x^3+1 \overline{)\,2x^5-8x^4+2x^3+x^2+0x+0} \\ -\;2x^5 \qquad\qquad\quad \mp x^2 \\ \hline -8x^4+2x^3 \qquad\qquad \\ +\;8x^4 \qquad\quad +4x \\ \hline 2x^3+0x^2+4x \\ -\;2x^3 \qquad\qquad \mp 1 \\ \hline 4x-1 \end{array}$$

$$\boxed{x^2-4x+1+\frac{4x-1}{2x^3+1}}$$

b. $\frac{x^5+x^3-2}{x-1}$

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 1 & 0 & 0 & -2 \\ & & 1 & 1 & 2 & 2 & 2 \\ \hline & 1 & 1 & 2 & 2 & 2 & \boxed{0} \end{array}$$

$$\boxed{x^4+x^3+2x^2+2x+2}$$

16. Write a polynomial with the given properties. You may leave the polynomial in factored form with real coefficients. (3 points each)

a. $n = 3$, zeros are $1, 5$ (multiplicity two) and $f(-1) = -104$

$a(x-1)(x-5)^2$

$a(-1-1)(-1-5)^2 = -104$

$a(-2)(36) = -104$

$a = \frac{-104}{-72} = \frac{13}{9}$

$f(x) = \frac{13}{9}(x-1)(x-5)^2$

b. $n = 4$, zeros $-2, 5, 3 + 2i$, and $f(1) = -96$

$(x-3-2i)(x-3+2i)$

$x^2 - 3x + \cancel{2xi} - 3x + 9 - \cancel{6i} - \cancel{2xi} + \cancel{6i} - 4i^2$

$= x^2 - 6x + 13$

$a(x+2)(x-5)(x^2-6x+13) =$

$a(1+2)(1-5)(1-6+13) = -96$

$a(3)(-4)(8) = -96$

$a(-96) = -96$

$a = 1$

$f(x) = (x+2)(x-5)(x^2-6x+13)$

17. Sketch the graph of the function $f(x) = \frac{x^3-1}{x^2-9}$, but finding i) any intercepts, ii) any vertical asymptotes or holes, iii) any horizontal or slant asymptotes. (5 points)

VA $x = 3, x = -3$

$$\begin{array}{r} x \\ x^2-9 \enclose{longdiv}{x^3 + 0x^2 + 0x - 1} \\ -\underline{x^3 + 9x} \\ 9x - 1 \end{array}$$

Slant asymptote $y = x$

$x + \frac{9x-1}{x^2-9}$

Intercepts $x = 1$

$y = \frac{1}{9}$

18. Solve the rational and polynomial inequalities and write the solution in interval notation. (4 points each)

a. $x^3 - x^2 + 9x - 9 > 0$

$x^2(x-1) + 9(x-1)$

$(x^2+9)(x-1) > 0$

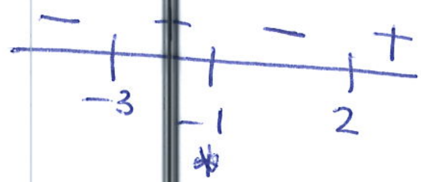
never zero

$-$ | $+$

1

$(1, \infty)$

b. $\frac{(x+3)(x-2)}{x+1} \leq 0$




$(-\infty, -3] \cup (-1, \infty)$

19. $y$ varies jointly as $m$ and the square of $n$, and inversely as $p$. $y = 15$ when $m = 2, n = 1, p = 6$. Find $y$ when $m = 3, n = 4, p = 10$. (5 points)

$$y = \frac{kmn^2}{p}$$

$$15 = \frac{k(2)(1)^2}{6}$$

$$k = 45$$

$$y = \frac{45mn^2}{6}$$

$$y = \frac{45(3)(16)}{10}$$

$$\boxed{y = 216}$$