MTH 166 Homework #10 Key



1\. a. $z = 4i$ $|z| = 4$

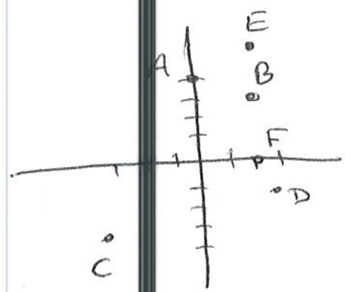
b. $2+3i$ $|z| = \sqrt{2^2+3^2} = \sqrt{13}$

c. $-3-4i$ $|z| = 5$

d. $3-i$ $|z| = \sqrt{10}$

e. $2+5i$ $|z| = \sqrt{29}$

f. $2$ $|z| = 2$




2a. $2+2i$ $r = \sqrt{8}$ $\theta = \pi/4$ $\sqrt{8}(\cos \pi/4 + i \sin \pi/4)$

b. $-2+2i\sqrt{3}$ $r = \sqrt{4+12} = 4$ $\theta = -\pi/3 + \pi = 2\pi/3$ $4(\cos 2\pi/3 + i \sin 2\pi/3)$

c. $-2+3i$ $r = \sqrt{13}$ $\theta \approx 2.16$ $\approx \sqrt{13}(\cos 2.16 + i \sin 2.16)$

d. $1 - i\sqrt{5}$ $r = \sqrt{6}$ $\theta \approx 5.13$ $\approx \sqrt{6}(\cos 5.13 + i \sin 5.13)$

3a. $6(\cos \pi/6 + i \sin \pi/6) = 6\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right) = 3\sqrt{3} + 3i$

b. $5(\cos \pi/2 + i \sin \pi/2) = 5(0 + i(1)) = 5i$

c. $8(\cos 7\pi/4 + i \sin 7\pi/4) = 8\left(1/\sqrt{2} + i\left(-\frac{1}{\sqrt{2}}\right)\right) = 4\sqrt{2} - 4\sqrt{2}i$

d. $20(\cos 205^\circ + i \sin 205^\circ) \approx 20(-.9063 + -.4226i) = -18.126 - 8.452i$

3a. $z_1 z_2 = 30(\cos 70^\circ + i \sin 70^\circ)$

$\frac{z_1}{z_2} = \frac{6}{5}(\cos(-30^\circ) + i \sin(-30^\circ))$

b. $z_1 z_2 = 12\left(\cos \frac{11\pi}{16} + i \sin 11\pi/16\right)$

$\frac{z_1}{z_2} = \frac{3}{4}(\cos 9\pi/16 + i \sin 9\pi/16)$

c. $z_1 z_2 = -2$ $\frac{z_1}{z_2} = \frac{1+i}{-1+i} = -i$

d. $z_1 z_2 = 5 - i$

$\frac{z_1}{z_2} = \frac{1+i}{2-3i} = -\frac{1}{13} + \frac{5}{13}i$

5a. $8(\cos 45° + i \sin 45°) = 4\sqrt{2} + 4\sqrt{2}i$

b. $\frac{1}{8}(\cos 5\pi/3 + i \sin 5\pi/3) = \frac{1}{8}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{1}{16} - \frac{\sqrt{3}}{16}i$

c. $(1+i)^4 = \left[\sqrt{2}(\cos \pi/4 + i \sin \pi/4)\right]^4 = -4$

$= 4(\cos \pi + i \sin \pi)$

d. $243(\cos 5\pi/4 + i \sin 5\pi/4) = -\frac{243}{\sqrt{2}} - \frac{243}{\sqrt{2}}i$

e. $(\sqrt{2} - i)^7 = -13\sqrt{2} + 43i$

6a. $3(\cos 5\pi/6 + i \sin 5\pi/6)$ and $3(\cos 11\pi/6 + i \sin 11\pi/6)$

b. $3(\cos 102° + i \sin 102°)$, $3(\cos 222° + i \sin 222°)$, $3(\cos 342° + i \sin 342°)$

c. $\sqrt{2}(\cos \pi/3 + i \sin \pi/3)$, $\sqrt{2}(\cos 5\pi/6 + i \sin 5\pi/6)$,
$\sqrt{2}(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$, $\sqrt{2}(\cos 11\pi/6 + i \sin 11\pi/6)$

d. $(1+i) = \sqrt{2}(\cos \pi/4 + i \sin \pi/4)$

$\sqrt[10]{2}(\cos \pi/20 + i \sin \pi/20)$, $\sqrt[10]{2}(\cos 9\pi/20 + i \sin 9\pi/20)$,
$\sqrt[10]{2}(\cos 17\pi/20 + i \sin 17\pi/20)$, $\sqrt[10]{2}(\cos 5\pi/4 + i \sin 5\pi/4)$,
$\sqrt[10]{2}(\cos 33\pi/20 + i \sin 33\pi/20)$

e. $1 = \cos 0 + i \sin 0 = \cos 2\pi + i \sin 2\pi = \cos 4\pi + i \sin 4\pi =$
$\cos 6\pi + i \sin 6\pi = \cos 8\pi + i \sin 8\pi = \cos 10\pi + i \sin 10\pi$

Cube roots: $\cos 0 + i \sin 0 = 1$
$\cos 2\pi/3 + i \sin 2\pi/3 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$
$\cos 4\pi/3 + i \sin 4\pi/3 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

fourth roots $1, i, -1, -i$

fifth roots: $\cos 0 + i \sin 0 = 1$, $\cos 2\pi/5 + i \sin 2\pi/5$,
$\cos 4\pi/5 + i \sin 4\pi/5$, $\cos 6\pi/5 + i \sin 6\pi/5$,
$\cos 8\pi/5 + i \sin 8\pi/5$

6e (cont'd) ③

Sixth roots: $\cos 0 + i\sin 0 = 1$, $\cos \pi/3 + i \sin \pi/3 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\cos 2\pi/3 + i \sin 2\pi/3 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$, $\cos \pi + i \sin \pi = -1$,

$\cos 4\pi/3 + i \sin 4\pi/3 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$, $\cos 5\pi/3 + i \sin 5\pi/3 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

f. $-i = \cos 3\pi/2 + i \sin 3\pi/2 = \cos 7\pi/2 + i \sin 7\pi/2 = \cos 11\pi/2 + i \sin 11\pi/2$

$= \cos 15\pi/2 + i \sin 15\pi/2 = \cos 19\pi/2 + i \sin 19\pi/2 = \cos 23\pi/2 + i \sin 23\pi/2$

Cube roots: $\cos \pi/2 + i \sin \pi/2 = i$, $\cos 7\pi/6 + i \sin 7\pi/6 = -\frac{\sqrt{3}}{2} - \frac{1}{2} i$

$\cos 11\pi/6 + i \sin 11\pi/6 = \frac{\sqrt{3}}{2} - \frac{1}{2} i$

fourth roots: $\cos 3\pi/8 + i \sin 3\pi/8$, $\cos 7\pi/8 + i \sin 7\pi/8$,

$\cos 11\pi/8 + i \sin 11\pi/8$, $\cos 15\pi/8 + i \sin 15\pi/8$

fifth roots: $\cos 3\pi/10 + i \sin 3\pi/10$, $\cos 7\pi/10 + i \sin 7\pi/10$,

$\cos 11\pi/10 + i \sin 11\pi/10$, $\cos 3\pi/2 + i \sin 3\pi/2$,

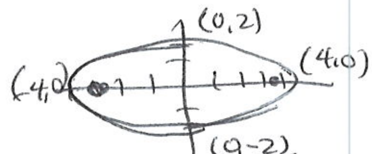
$\cos 19\pi/10 + i \sin 19\pi/10$

sixth roots: $\cos \pi/4 + i \sin \pi/4$, $\cos 7\pi/12 + i \sin 7\pi/12$,

$\cos 11\pi/12 + i \sin 11\pi/12$, $\cos 15\pi/12 + i \sin 15\pi/2$

$\cos 19\pi/12 + i \sin 19\pi/12$, $\cos 23\pi/12 + i \sin 23\pi/12$

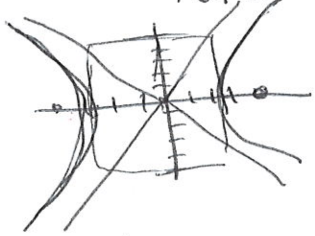
7\. a. $\frac{x^2}{16} + \frac{y^2}{4} = 1$




$c = 16 - 4 = \sqrt{12}$

$(\sqrt{12}, 0)$ $(-\sqrt{12}, 0)$ foci

b. $\frac{x^2}{8} - \frac{y^2}{25} = 1$



$8 + 25 = 31$

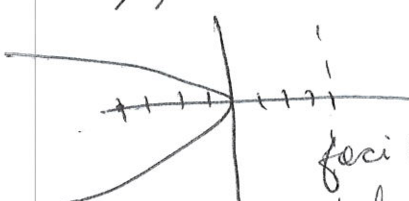
$c = \sqrt{31}$

foci $(\pm\sqrt{31}, 0)$

vertices $(\pm\sqrt{8}, 0)$

asymptotes $y = \frac{\pm 5}{2\sqrt{2}} x$

c. $y^2 = -8x$

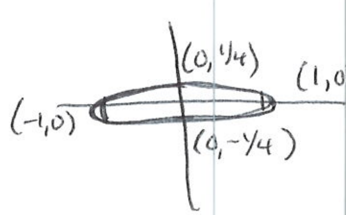


foci $(-4, 0)$

directrix $x = 4$

d. $x^2 = 1 - 4y^2$

$x^2 + 4y^2 = 1 \Rightarrow x^2 + \frac{y^2}{(1/4)} = 1$

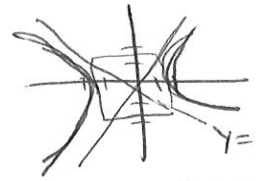



$1 - 1/4 = \frac{\sqrt{3}}{2}$

foci $(\pm\frac{\sqrt{3}}{2}, 0)$

7e. $y^2 = x^2 - 3$

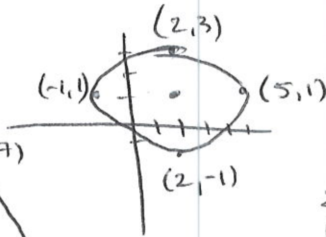
$y^2 - x^2 = -3 \Rightarrow x^2 - y^2 = 3 \Rightarrow \frac{x^2}{3} - \frac{y^2}{3} = 1$




vertices $(\pm\sqrt{3}, 0)$

foci $(\pm\sqrt{6}, 0)$

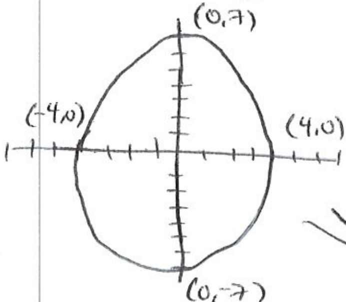
f. $\frac{(x-2)^2}{9} + \frac{(y-1)^2}{4} = 1$




center $(2,1)$

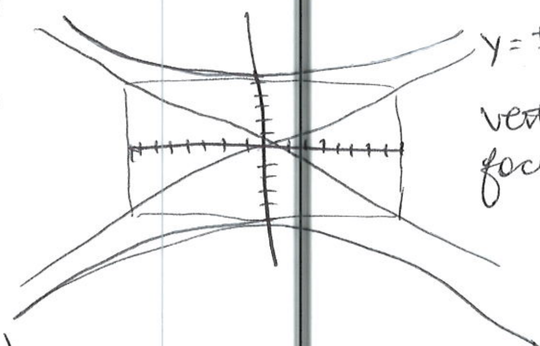
foci $(2 \pm \sqrt{5}, 1)$

m. $\frac{x^2}{16} + \frac{y^2}{49} = 1$




foci $(0, \pm\sqrt{33})$

n. $\frac{y^2}{25} - \frac{x^2}{64} = 1$



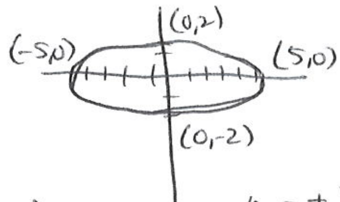
$y = \pm\frac{5}{8}x$

vertices $(0, \pm 5)$

foci $(0, \pm\sqrt{89})$

o. $\frac{4x^2}{100} + \frac{25y^2}{100} = \frac{100}{100}$

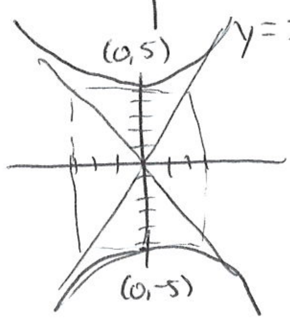
$\frac{x^2}{25} + \frac{y^2}{4} = 1$




foci $(\pm\sqrt{21}, 0)$

p. $\frac{9y^2}{225} - \frac{25x^2}{225} = \frac{225}{225}$

$\frac{y^2}{25} - \frac{x^2}{9} = 1$



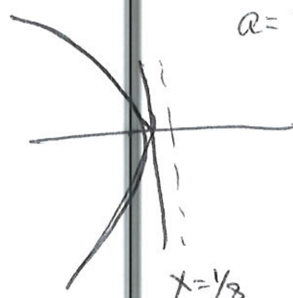

$y = \pm\frac{5}{3}x$

foci $(0, \pm\sqrt{34})$

q. $8y^2 + 4x = 0$

$y^2 = -\frac{1}{2}x$

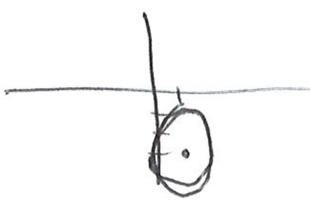
$a = \frac{1}{8}$




vertex $(0,0)$

focus $(-1/8, 0)$

r. $\frac{(x-1)^2}{2} + \frac{(y+3)^2}{5} = 1$



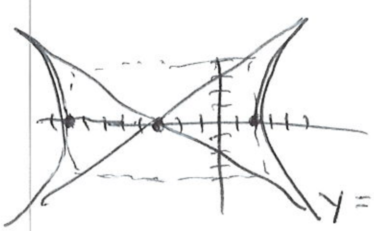
Center $(1,-3)$

vertex $(1, -3 \pm \sqrt{5})$

minor endpts $(1 \pm \sqrt{2}, -3)$

foci $(1, -3 \pm \sqrt{3})$

g. $\frac{(x+3)^2}{25} - \frac{y^2}{16} = 1$

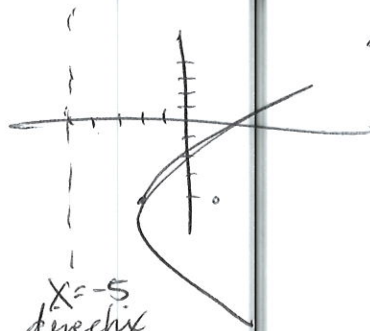


$y = \pm\frac{4}{5}(x+3)$

center $(-3,0)$

vertices $(2,0), (-8,0)$

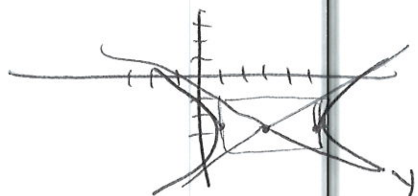
7h. $(y+4)^2 = 12(x+2)$




vertex $(-2,-4)$
$a=3$
focus $(1,-4)$

i. $\frac{(x-3)^2}{4} - \frac{4(y+3)^2}{4} = \frac{4}{4}$

$$\frac{(x-3)^2}{4} - \frac{(y+3)^2}{1} = 1$$



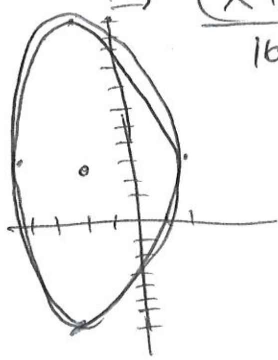
center $(3,-3)$
vertices $(5,-3)(1,-3)$
foci $(3\pm\sqrt{5},-3)$
$y+3 = \pm\frac{1}{2}(x-3)$

j. $4x^2+y^2+16x-6y-39=0$

$4x^2+16x + y^2-6y = 39$

$4(x^2+4x+4)+(y^2-6y+9) = 39+16+9$

$\frac{4(x+2)^2}{64} + \frac{(y-3)^2}{64} = 64 \Rightarrow \frac{(x+2)^2}{16} + \frac{(y-3)^2}{64} = 1$



center $(-2,3)$
focus $(-2, 3\pm4\sqrt{3})$
vertices $(-2,11), (-2,-5)$
minor endpt $(2,3), (-6,3)$

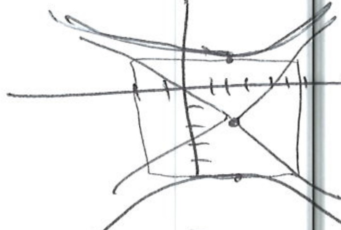
k. $9x^2-16y^2-36x-64y+116=0$

$9x^2-36x \quad -16y^2-64y = -116$

$9(x^2-4x+4)-16(y^2+4y+4) = -116+36-64 = \frac{-144}{-144}$

$\frac{9(x-2)^2}{-144} - \frac{16(y+2)^2}{-144} = 1 \Rightarrow \frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$

center $(2,-2)$
foci $(2,3), (2,-7)$
vertices $(2,1), (2,-5)$



$y+2 = \pm\frac{3}{4}(x-2)$

l. $y^2-2y+12x-35=0$

$(y^2-2y+1) = -12x+35+1$

$(y-1)^2 = -12(x-3)$


vertex $(3,1)$
focus $(0,1)$
directrix $x=6$
$a=3$

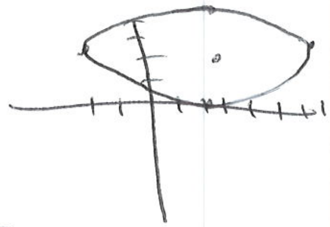
6

75. $(x-2)^2 = 8(y-1)$


vertex $(2,1)$
focus $(2,3)$

t. $\frac{(x-3)^2}{16} + \frac{4(y-2)^2}{16} = \frac{16}{16}$

$\frac{(x-3)^2}{16} + \frac{(y-2)^2}{4} = 1$



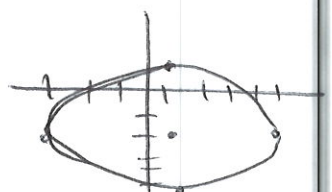
center $(3,2)$
vertices $(7,2)$ $(-1,2)$
minor endpt $(3,4)$, $(3,0)$
foci $(3 \pm 2\sqrt{3}, 2)$

u. $9x^2 + 16y^2 - 18x + 64y - 71 = 0$

$9x^2 - 18x + 16y^2 + 64y = 71$

$9(x^2 - 2x + 1) + 16(y^2 + 4y + 4) = 71 + 9 + 64$

$\frac{9(x-1)^2}{144} + \frac{16(y+2)^2}{144} = \frac{144}{144} \Rightarrow \frac{(x-1)^2}{16} + \frac{(y+2)^2}{9} = 1$



center $(1,-2)$
vertices $(-3,-2)$, $(5,-2)$
minor endpt $(1,1)$ $(1,-5)$
foci $(1 \pm \sqrt{7}, -2)$

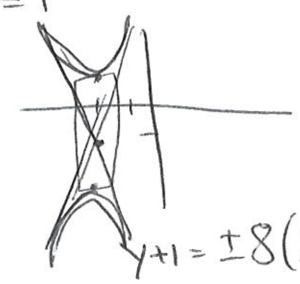
v. $16x^2 - y^2 + 64x - 2y + 67 = 0$

$16x^2 + 64x - y^2 - 2y = -67$

$16(x^2 + 4x + 4) - (y^2 + 2y + 1) = -67 + 64 - 1$

$\frac{16(x+2)^2}{-4} - \frac{(y+1)^2}{-4} = \frac{-4}{-4} \Rightarrow \frac{(y+1)^2}{4} - \frac{(x+2)^2}{(1/4)} = 1$

center $(-2,-1)$
vertex $(-2,1)$ $(-2,-3)$
foci $\left(-2, -1 \pm \frac{\sqrt{17}}{2}\right)$



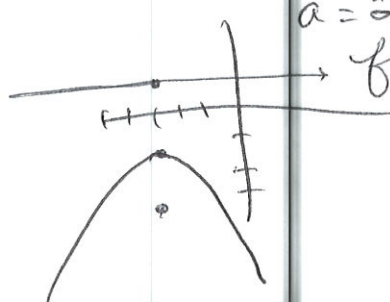
$y+1 = \pm 8(x+2)$

w. $x^2 + 6x + 8y + 1 = 0$

$x^2 + 6x + 9 = -8y - 1 + 9$

$(x+3)^2 = -8(y+1)$

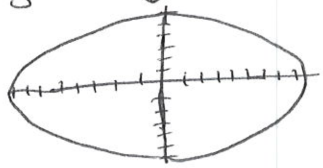
Center = vertex = $(-3,-1)$
$a = 2$
focus $(-3,-3)$
directrix $y = 1$



8.

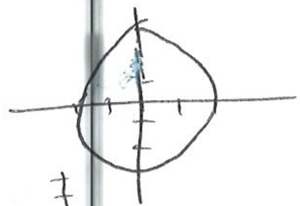
a. center $(0,0)$ $c=5$, $a=8$ $64-25=39$ $b=\sqrt{39}$

$$\frac{x^2}{64}+\frac{y^2}{39}=1$$



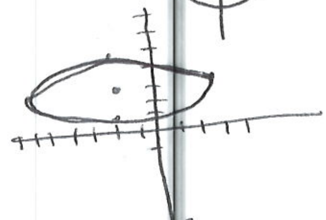
b. $b=2$, $c=2$ $a=\sqrt{8}$ center $(0,0)$

$$\frac{x^2}{4}+\frac{y^2}{8}=1$$



c. $a=5$, $b=2$ center $(-2,3)$

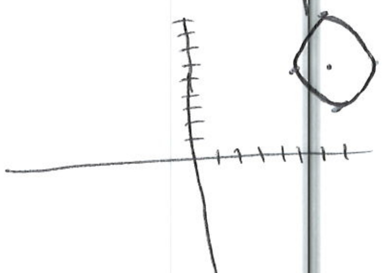
$$\frac{(x+2)^2}{25}+\frac{(y-3)^2}{4}=1$$



d. center $(7,6)$

$a=3$, $b=2$

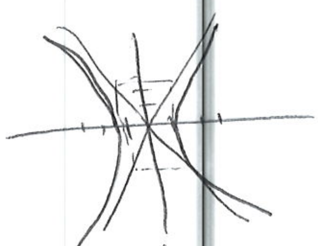
$$\frac{(x-7)^2}{4}+\frac{(y-6)^2}{9}=1$$



e. center $(0,0)$

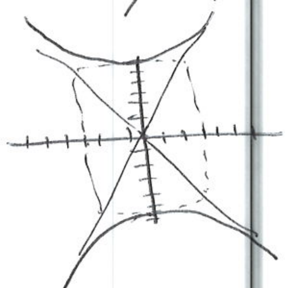
$c=3$, $a=1$ $9-1=8$ $b=\sqrt{8}$

$$\frac{x^2}{1}-\frac{y^2}{8}=1$$



f. $a=6$ $2=\frac{6}{b}\Rightarrow b=3$

$$\frac{y^2}{36}-\frac{x^2}{9}=1$$



g. center $(4,-2)$

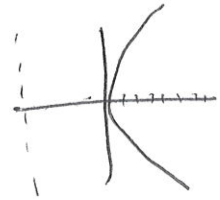
$c=3$, $a=2$

$9-4=5$ $b=\sqrt{5}$

$$\frac{(x-4)^2}{4}-\frac{(y+2)^2}{5}=1$$

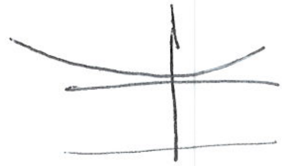
8h. vertex (0,0) $a=7$

$$y^2 = 28x$$



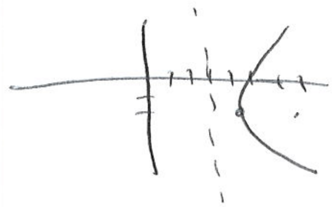
i. center (0,0) $a=15$

$$x^2 = 60y$$



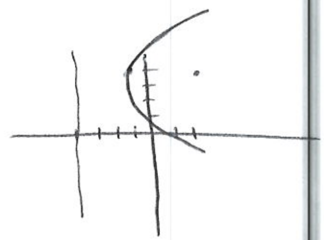
j. vertex (5,-2) $a=2$

$$(y+2)^2 = 8(x-5)$$



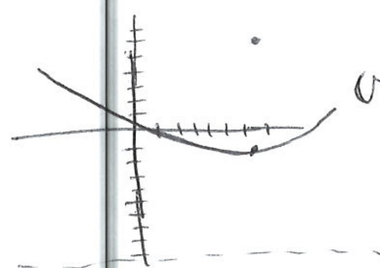
k. vertex (-1,4) $a=3$

$$(y-4)^2 = 12(x+1)$$



l. $a=8$

$$(x-7)^2 = 32(y+1)$$





9a. center (-1,2) $a=1$

$$(x+1)^2 = 4(y-2)$$

b. Center (0,0) $a=2$

$$y^2 = -8x$$

c. center (0,0)

$a=4$ $c=\sqrt{41}$ $b=5$

$$\frac{x^2}{16} - \frac{y^2}{25} = 1$$

d. $(x-2)^2 + (y-3)^2 = 17$

circle

e. $a=5$ $b=4$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

f. Center (-3,5)

$a=5$ $b=3$

$$\frac{(x+3)^2}{25} + \frac{(y-5)^2}{9} = 1$$

g. center (0,0)

$a=3$ $b=2$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

10a. $b=23$ $a=48$ $\quad \frac{x^2}{48^2} + \frac{y^2}{23^2} = 1$

b. focus $c = 42.1$ ft.

(9)

11a. $\frac{x^2}{5000^2} + \frac{y^2}{4750^2} = 1$ focus $(16,0)$

$\approx 750$ miles perigee

apogee $\approx 1016$ miles

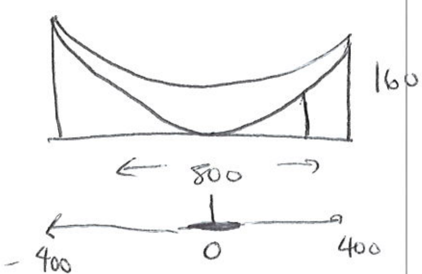
12.


$y = \frac{1}{2}x$ $\quad \frac{a}{b} = \frac{1}{2} = \frac{3}{b}$ $\quad b = 6$

$\frac{b}{a} = \frac{1}{2} = \frac{b}{3} \rightarrow b = \frac{3}{2}$

$\frac{x^2}{9} - \frac{y^2}{(3/2)^2} = 1$ or $\frac{y^2}{9} - \frac{x^2}{36} = 1$

13.




$(400, 160)$

$y = ax^2$

$160 = a(400)^2$

$a = .001$

$y = .001x^2$

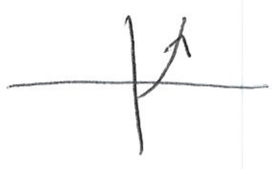
$y = .001(100)^2$

$y = 10$ feet

14. a.

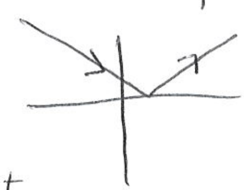
$x - 2 = t \quad y = (x-2)^2$

b. $x^2 = t \quad y = x^2 - 1$



c. $x = 2t \Rightarrow \frac{x}{2} = t$

$y = \left|\frac{x}{2} - 1\right|$

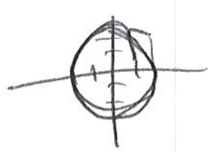


d. $x = 2\sin t$, $y = 2\cos t$

$x^2 + y^2 = 4$

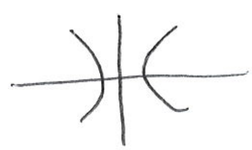
e. $x = 2\cos t \quad y = 3\sin t$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$



f. $x = \sec t$, $y = \tan t$

$x^2 - y^2 = 1$



15a. $x=t \quad y=4x-3$

b. $x=t,\ y=t^2-3$

c. $x=6\cos t+3,\ y=6\sin t+5$

d. $x=5\cos t-2,\ y=2\sin t+3$

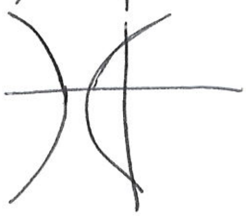
e. $a=4,\ c=5 \quad b=3$

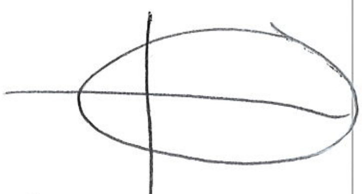
$x=4\tan t,\ y=3\sec t$

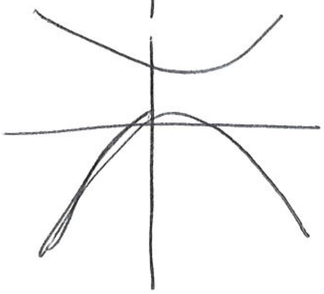
f. $\Delta x=3 \quad \Delta y=3$

$x=3t-2,\ y=3t+4$

16. a.  parabola $e=1$

b.  hyperbola $e=2 > 1$

c.  ellipse $e=2/3 < 1$

d.  hyperbola $e=\frac{5}{4} > 1$