MTH 166 Homework #2 Key

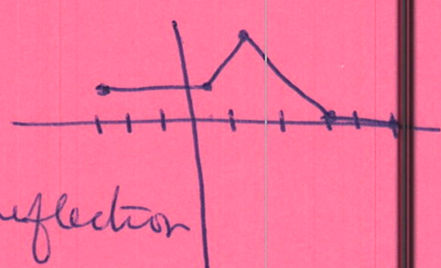
1a. key points $(-6,1), (1,1), (3,3), (6,0), (10,0)$

i. $f(x+1)$ left shift by 1

points $(-7,1), (0,1), (2,3), (5,0), (9,0)$

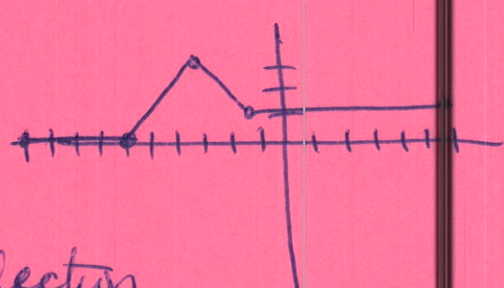
ii. $f(2x)$ compress by 2 $(1/2)$ horizontal

points $(-3,1), (1/2,1), (3/2,3), (3,0), (5,0)$



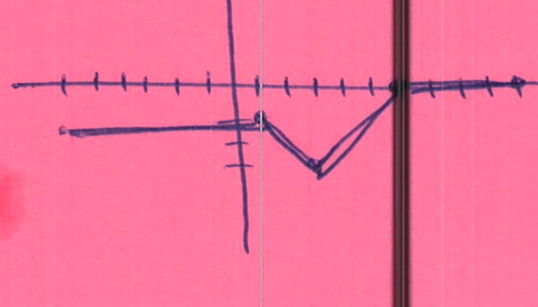
iii. $f(-x)$ horizontal reflection

points $(6,1), (-1,1), (-3,3), (-6,0), (-10,0)$



iv. $-f(x)$ vertical reflection

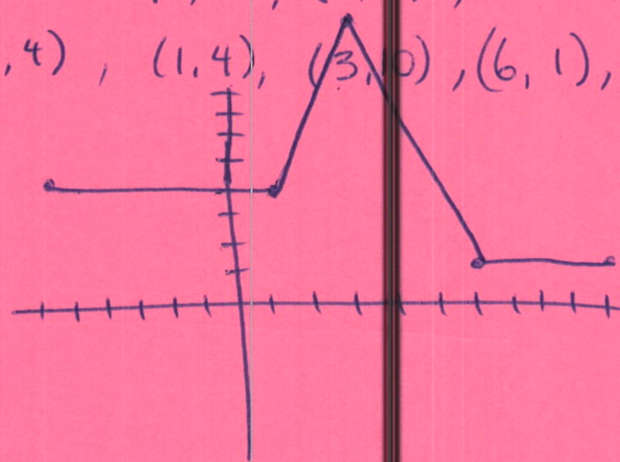
points $(-6,-1), (1,-1), (3,-3), (6,0), (10,0)$



1a. v. $3f(x)+1$ vertical stretch + vertical shift up 1

points stretch $(-6,3)$, $(1,3)$, $(3,9)$, $(6,0)$, $(10,0)$

+1 $(-6,4)$, $(1,4)$, $(3,10)$, $(6,1)$, $(10,1)$



vi. $-\frac{1}{2}f(x-3)-2$ horizontal shift +2, vertical reflection, vertical compression ½, vertical shift down 2

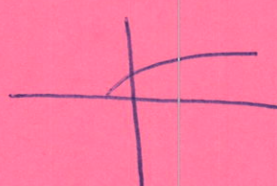
points H. shift $(-4,1)$, $(3,1)$, $(5,3)$, $(8,0)$, $(12,0)$

v. refl + comp $(-4,-\frac{1}{2})$, $(3,-\frac{1}{2})$, $(5,-\frac{3}{2})$, $(8,0)$, $(12,0)$

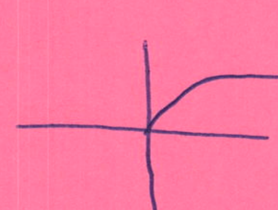
shift down $(-4,-\frac{5}{2})$, $(3,-\frac{5}{2})$, $(5,-\frac{7}{2})$, $(8,-2)$, $(12,-2)$

b. $f(x)=\sqrt{x}$

i. $f(x+1)=\sqrt{x+1}$

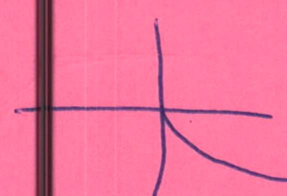


ii. $f(2x)=\sqrt{2x}$

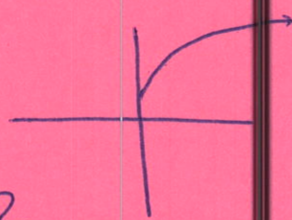


iii $f(-x)=\sqrt{-x}$

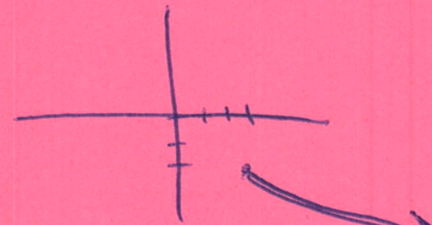
iv. $-f(x)=-\sqrt{x}$



1b. v. $3f(x)+1 = 3\sqrt{x}+1$

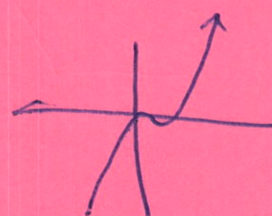


vi. $-\frac{1}{2}f(x-3)-2 = -\frac{1}{2}\sqrt{x-3}-2$

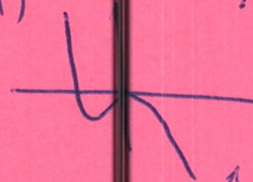


1c. i. $f(x+1) = (x+1)^2(x)$

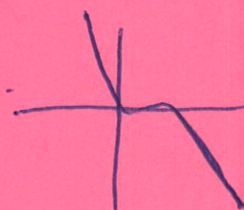
ii. $f(2x) = 4x^2(2x-1)$



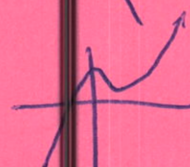
iii. $f(\cancel{x}) = (-x)^2(-x-1) = -x^2(x+1)$



iv. $-f(x) = -x^2(x-1)$



v. $3f(x)+1 = 3x^2(x-1)+1$



vi. $-\frac{1}{2}f(x-3)-2 = -\frac{1}{2}(x-3)^2(x-4)-2$

2a. $f(x)=(x-2)^2$ right shift 2

b. $f(x)=\sqrt{-x+3}=\sqrt{-(x-3)}$ horizontal reflection, right shift 3

c. $f(x) = -|x+4|+1$ vertical reflection, left shift 4, up shift 1

d. $f(x) = \frac{1}{2}(x+1)^3-4$ left shift 1, vertical compression ½, down shift 4.

3. $f(x) = 5-x^2$, $g(x) = 6-\frac{1}{x}$, $h(x)=\sqrt{2-x}$

all $\mathbb{R}$ $\quad x\neq 0 \quad 2-x\geq 0 \Rightarrow x\leq 2$

a. $f+g = 5-x^2+6-\frac{1}{\cancel{x}} = 11-x^2-\frac{1}{x}$ D: $x\neq 0$

b. $g-h = 6-\frac{1}{x}-\sqrt{2-x}$ D: $(-\infty,0)\cup(0,2]$

c. $fh = (5-x^2)\sqrt{2-x}$ D: $x\leq 2$

3d. $\frac{g}{f} = \frac{6-\frac{1}{x}}{5-x^2}\cdot\frac{x}{x} = \frac{6x-1}{5x-x^3}$ D: $x\neq 0,\ x\neq \pm\sqrt{5}$

e. $h\circ g = \sqrt{2-(6-\frac{1}{x})} = \sqrt{\frac{1}{x}-4}$

$\frac{1}{x} - 4 \geq 0$

$1-4x\geq 0$

$1\geq 4x \Rightarrow x\leq \frac{1}{4}$

D: $(-\infty,0)\cup(0,1/4)$

f. $h\circ h = \sqrt{2-\sqrt{2-x}}$

$2-\sqrt{2-x}\geq 0$

$2\geq \sqrt{2-x}$

$4\geq 2-x$

$x\geq -2$

D: $[-2,2]$

g. $f\circ h = 5-(\sqrt{2-x})^2 = 5-(2-x) = 3+x$ D: $x\leq 2$

h. $f\circ g\circ h = 5-\left(6-\frac{1}{\sqrt{2-x}}\right)^2$

$5-\left(36-\frac{12}{\sqrt{2-x}}-\frac{1}{2-x}\right)$

$= -31+\frac{12}{\sqrt{2-x}}-\frac{1}{2-x}$

D: $x\neq 0,\ x<2$

$(-\infty,0)\cup(0,2]$

4a. $h(x)=(x^2+2x-1)^4$ $f(x)=x^4,\ g(x)=x^2+2x-1$

b. $h(x)=\sqrt[3]{7x+4}$ $f(x)=\sqrt[3]{x},\ g(x)=7x+4$

c. $h(x)=\frac{|2x+3|}{2x-3}$ $f(x)=\frac{|x+6|}{x}$ $g(x)=2x-3$

5a. $f(x)=2x+3$

$x=2y+3$

$\frac{x-3}{2}=y=f^{-1}(x)$

5b. $f(x) = x^3 - 1$

$x = y^3 - 1$

$\sqrt[3]{x+1} = y = f^{-1}(x)$

c. $f(x) = \dfrac{2x+1}{x-3}$

$x = \dfrac{2y+1}{y-3}$

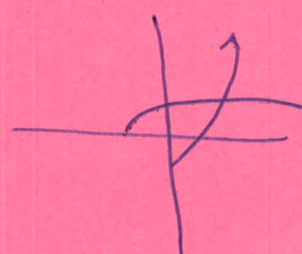
$xy - 3x = 2y + 1$

$xy - 2y = 3x + 1$

$y(x-2) = 3x + 1$

$f^{-1} = y = \dfrac{3x+1}{x-2}$

6. $f(x) = x^2 - 1$ D(restricted) $[0, \infty)$



$x = y^2 - 1$

$x + 1 = y^2$

$\sqrt{x+1} = y = f^{-1}(x)$

7. $\left(-\frac{1}{4}, \frac{1}{7}\right), \left(\frac{3}{4}, \frac{6}{7}\right)$ $d = \sqrt{\left(-\frac{1}{4} - \frac{3}{4}\right)^2 + \left(\frac{1}{7} - \frac{6}{7}\right)^2} =$

$\sqrt{(-1)^2 + \left(-\frac{5}{7}\right)^2} = \sqrt{1 + \frac{25}{49}} =$

$\sqrt{\frac{49+25}{49}} = \sqrt{\frac{74}{49}} = \frac{\sqrt{74}}{7}$

8a. $(x^2 + 6x + 9) + (y^2 + 2y + 1) = -6 + 9 + 1$

$(x+3)^2 + (y+1) = 4$

Center $(-3, -1)$, $R = 2$

8b. $(x^2+12x+36)+(y^2-6y+9)=4+36+9$

$$(x+6)^2+(y-3)^2=49$$

center $(-6,3)$, radius $=7$

8c. $\left(x^2+3x+\frac{9}{4}\right)+\left(y^2+5y+\frac{25}{4}\right)=-\frac{9}{4}+\frac{9}{4}+\frac{25}{4}$

$\left(\frac{3}{2}\right)^2 \qquad \left(\frac{5}{2}\right)^2$

$$(x+3/2)^2+(y+5/2)^2=\frac{25}{4}$$

Center $(-3/2,-5/2)$ radius $=5/2$

9.

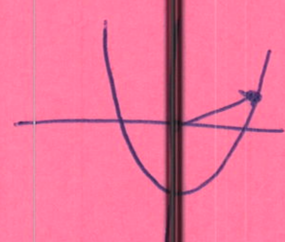
$V(x)=(24-2x)(24-2x)x = (24-2x)^2x$

$V(2)=2(20)^2=800$

$V(6)=6(12)^2=864$

D: $(0,12)$

10. $d=\sqrt{(0-x)^2+(0-(x^2-4))^2}$



$=\sqrt{x^2+(x^2-4)^2}$

$=\sqrt{x^2+x^4-8x^2+16}$

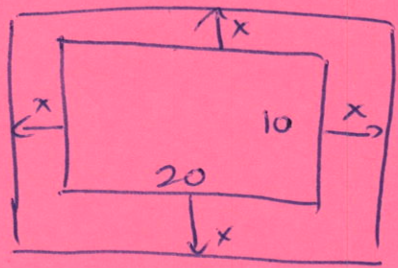
$=\sqrt{x^4-7x^2+16}$

local max at $x=0$ $\quad d=4$

local min at $x\approx\pm1.87$

$d\approx1.93$

11.



$$A_{TOTAL} = (20+2x)(10+2x) - 200$$
$$= \cancel{200} + 40x + 20x + 4x^2 - \cancel{200}$$
$$= 4x^2 + 60x$$

12. a. $7-(-9+2i)-(-17-i) = -7+9-2i+17+i = 19-i$

b. $(2+3i)^2 = 4+12i+\underbrace{9i^2}_{-9} = -5+12i$

c. $-\dfrac{6i}{3+2i}\cdot\dfrac{3-2i}{3-2i} = \dfrac{-18i+12i^2}{9+4} = \dfrac{-12-18i}{13} = -\dfrac{12}{13}-\dfrac{18}{13}i$

d. $5\underbrace{\sqrt{8}}_{2\sqrt2}i + 3\underbrace{\sqrt{18}}_{3\sqrt2}i = 10\sqrt{2}i + 9\sqrt{2}i = 19\sqrt{2}i$

e. $(3\sqrt{5}i)(-4\sqrt{12}i) = -12i^2\sqrt{60} = 24\sqrt{15}$

f. $\dfrac{1+i}{2+i}+\dfrac{1-i}{2-i} = \dfrac{(1+i)(2-i)}{5}+\dfrac{(1-i)(2+i)}{5} = \dfrac{2\cancel{-i}+\cancel{2i}+1+2\cancel{+i}-\cancel{2i}+1}{5} = \dfrac{5}{5}$

$=1$

13 a. $x = \dfrac{6\pm\sqrt{36-40}}{2} = \dfrac{6\pm 2i}{2} = 3\pm i$

b. $3x^2-4x+6=0$

$$x = \frac{4\pm\sqrt{16-4(3)(6)}}{6} = \frac{4\pm\sqrt{-56}}{6} = \frac{4\pm 2\sqrt{14}i}{6} = \frac{2}{3}\pm\frac{\sqrt{14}}{3}i$$

14. a. $3x^2-12x+1 = f(x)$

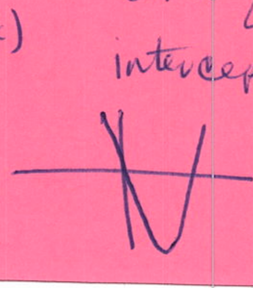
$3(x^2-4x+4)+1-12 = f(x)$

$3(x-2)^2-11 = f(x)$

vertex $(2,-11)$

axis of symmetry $x=2$

intercepts x: $\dfrac{12\pm\sqrt{144-12}}{6} = \dfrac{12\pm2\sqrt{33}}{6}$



$= 2\pm\dfrac{\sqrt{33}}{3}$ and, $y=1$

14b. $f(x) = \frac{5}{4} - (x - \frac{1}{2})^2$

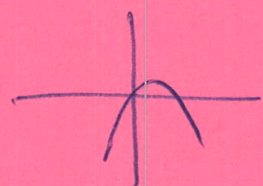
vertex $(1/2, 5/4)$

axis of symmetry $x = \frac{1}{2}$

intercepts $y = 1$

$\frac{5}{4} - (x^2 - x + \frac{1}{4}) = -x^2 + x + 1$

$x = \frac{-1 \pm \sqrt{1+4}}{-2} = \frac{-1 \pm \sqrt{5}}{-2} = \frac{1 \pm \sqrt{5}}{2}$



c. $f(x) = -2x^2 - 12x + 3$

$f(x) = -2(x^2 + 6x + 9) + 3 + 18$

$-2(x+3)^2 + 21$

intercepts

$y = 3$

$x = \frac{-12 \pm \sqrt{144 + 24}}{-6} = \frac{-12 \pm \sqrt{168}}{-6}$

$2 \pm \frac{\sqrt{42}}{3}$

vertex $(-3, 21)$ axis of symmetry $x = -3$