TH 166 Homework #4 Key

1a. $2^{3.4} = 10.5561$

c. $e^{2.3} = 9.9742$

e. $4^{-1.5} = .125$

g. $e^{-.95} = .3867$

i. $(1/3)^{1.7} = .1545$

k. $(\ln 2)^{-1.8} = 1.9343$

b. $\log_{15} 13 = \frac{\ln 13}{\ln 15} = .9472$

d. $\log_{\pi} 63 = \frac{\ln 63}{\ln \pi} = 3.6193$

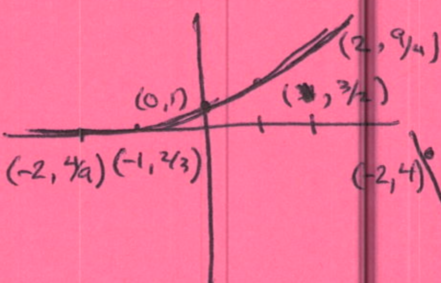
f. $\log_{0.3} 19 = \frac{\ln 19}{\ln 0.3} = -2.4456$

h. $\log_{16} 57.2 = \frac{\ln 57.2}{\ln 16} = 1.4595$

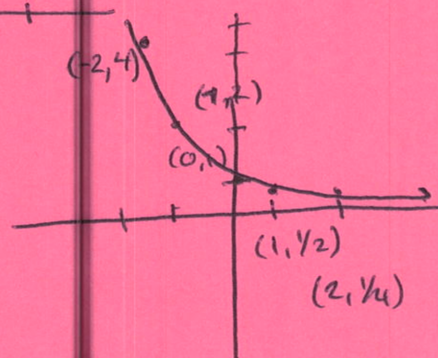
j. $\ln(11) = 2.3979$

l. $\log_{1/6} 99 = \frac{\ln 99}{\ln(1/6)} = -2.5646$

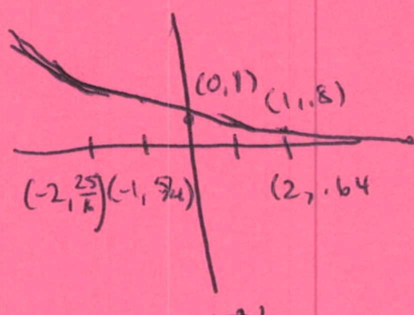
2. a. $f(x) = \left(\frac{3}{2}\right)^x$



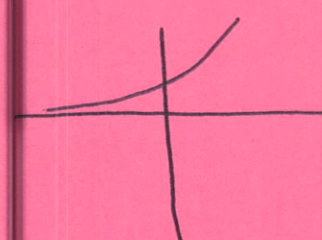
b. $g(x) = (1/2)^x$



c. $h(x) = 0.8^x$

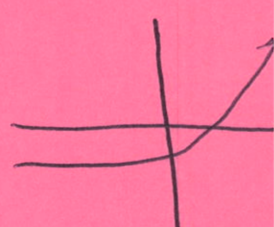


3a. $f(x) = 2^x \qquad f(x+1) = 2^{x+1}$

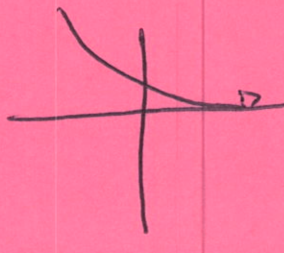


b. $-f(x) - 1 = -2^x - 1$

c. $3f(x-1) - 4 = 3(2^{x-1}) - 4$

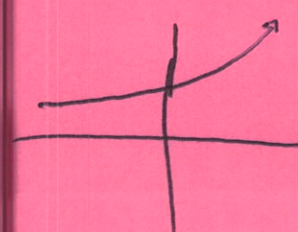


d. $g(x) = 2^{-x}$



e. $g(x) = \frac{1}{2}f(x-2) + 3$

$= \frac{1}{2}(2^{x-2}) + 3$



3a. inverses

$x = 2^{y+1}$

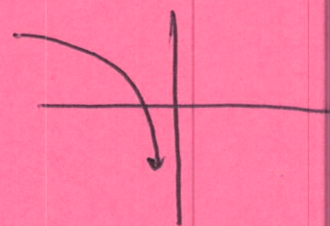
$\log_2(x) - 1 = y$

b. $x = -2^y - 1$

$x + 1 = -2^y$

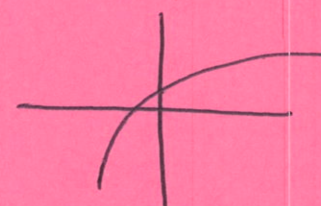
$-(x+1) = 2^y$

$\log_2(-(x+1)) = y$



c. $x = 3(2^{y-1}) - 4$

$\frac{(x+4)}{3} = 2^{y-1} \rightarrow \log_2\left(\frac{x+4}{3}\right) + 1 = y$



d. $x = 2^{-y} \rightarrow -\log_2 x = y$

e. $x = \frac{1}{2}(2^{y-2}) + 3$

$x - 3 = \frac{1}{2}(2^{y-2})$

$2(x-3) = 2^{y-2}$

$\log_2(2(x-3)) + 2 = y$

4a. i. $\left(1+\frac{.055}{12}\right)^{12(5)} = 1.3157 \times$ original investment

ii. $\left(1+\frac{.055}{52}\right)^{52(5)} = 1.3163 \times$ original investment

iii. $(1+.055)^{5} = 1.30696 \times$ original investment

iv. $\left(1+\frac{.055}{365}\right)^{365(5)} = 1.31650 \times$ original investment

v. $e^{.055(5)} = 1.31653 \times$ original investment

b. i. $12{,}000 = P\left(1+\frac{.0825}{1}\right)^{4} \Rightarrow \frac{12{,}000}{\left(1+\frac{.0825}{1}\right)^{4}} = \$8739.16$

ii. $\frac{12{,}000}{\left(1+\frac{.0825}{12}\right)^{12(4)}} = \$8636.83$

iii. $\frac{12{,}000}{\left(1+\frac{.0825}{365}\right)^{365(4)}} = \$8627.41$

iv. $\frac{12{,}000}{e^{.0825(4)}} = \$8627.08$

5a. $\log_4 16 = \log_4 4^2 = 2$

b. $\log_2 \frac{1}{\sqrt{2}} = \log_2 2^{-1/2} = -1/2$

c. $8^{\log_8 19} = 19$

d. $\log_6 (1) = 0$

e. $\log_5 5 = 1$

f. $\log_3 \frac{1}{9} = \log_3 3^{-2} = -2$

g. $\log_4 4^6 = 6$

h. $e^{\ln(11)} = 11$

6a. $f(x) = 2 + \log_3 x$

$x = 2 + \log_3 y$

$3^{(x-2)} = y$

$(0, 1/9)$ $(3, 3)$
$(1, 1/3)$ $(4, 9)$
$(2, 1)$

$(1/9, 0)$ $(3, 3)$
$(1/3, 1)$ $(9, 4)$
$(1, 2)$

7a. $f(x) = 3e^x - 2$

D: all reals R: $[-2, \infty)$

b. $h(x) = \frac{1}{2}\ln x$

D: $(0, \infty)$ R: all reals

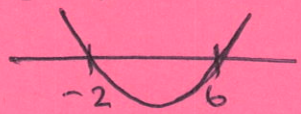
c. $m(x) = \ln(x^2 - 4x - 12)$

$x^2 - 4x - 12 > 0$

$(x-6)(x+2) > 0$

D: $(-\infty, -2) \cup (6, \infty)$

R: all reals



d. $g(x) = -2(1/3)^{x+2} + 5$

D: all reals R: $(-\infty, 5)$

e. $k(x) = 2 - \ln(7 - x)$

D: $(-\infty, 7)$ R: all reals

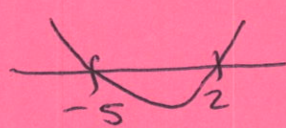
$7 - x > 0$

$7 > x$

f. $n(x) = \ln\left(\frac{x-2}{x+5}\right)$

D: $(-\infty, -5) \cup (2, \infty)$

$\frac{x-2}{x+5} > 0$



R: all reals

8.a. $\log_4 x = -3 \rightarrow x = 4^{-3} = \frac{1}{64}$

b. $\log_{64} x = \frac{2}{3} \rightarrow x = 64^{2/3} = (\sqrt[3]{64})^2 = 4^2 = 16$

c. $\log_5(x+4) = 2 \rightarrow x + 4 = 5^2 = 25 \rightarrow x = 21$

9a. $\log_9\left(\frac{9}{x}\right) = \log_9 9 - \log_9 x = 1 - \log_9 x$

b. $\ln \sqrt[7]{x} = 1/7 \ln x$

5

9c. $\log_8\left(\frac{64}{\sqrt{x+1}}\right) = \log_8 64 - \log_8 \sqrt{x+1} = \log_8 8^2 - \frac{1}{2}\log_8(x+1)$

$= 2 - \frac{1}{2}\log_8(x+1)$

d. $\log_b\left(\frac{\sqrt{x}\,y^3}{z^3}\right) = \log_b\sqrt{x} + \log_b y^3 - \log_b z^3 =$

$\frac{1}{2}\log_b x + 3\log_b y - 3\log_b z$

e. $\ln\sqrt{ex} = \frac{1}{2}\ln(ex) + \frac{1}{2}[\ln e + \ln x] = \frac{1}{2}[1 + \ln x]$

f. $\log_2 \sqrt[5]{\frac{xy^4}{16}} = \frac{1}{5}\log_2 \frac{xy^4}{16} = \frac{1}{5}[\log_2 x + \log_2 y^4 - \log_2 16]$

$= \frac{1}{5}[\log_2 x + 4\log_2 y - 4]$

g. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right] = 4\ln x + \frac{1}{2}\ln(x^2+3) - 5\ln(x+3)$

h. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right] = \log 100 + \log x^3 + \log\sqrt[3]{5-x} - \log 3$

$- \log(x+7)^2 =$

$2 + 3\log x + \frac{1}{3}\log(5-x) - \log 3 - 2\log(x+7)$

10a. $\log 5 + \log 2 = \log 10 = 1$

b. $4\ln(x+6) - 3\ln z = \ln\left[\frac{(x+6)^4}{z^3}\right]$

c. $\frac{1}{3}[\log_4 x - \log_4 y] = \log_4\sqrt[3]{\frac{x}{y}}$

d. $\frac{1}{2}[\log_5 x + \log_5 y] - 2\log_5(x+1) = \log_5\left[\frac{\sqrt{xy}}{(x+1)^2}\right]$

e. $\log x + \log(x^2-1) - \log 7 - \log(x+1) = \log\left[\frac{x(x^2-1)}{7(x+1)}\right] = \log\left[\frac{x(x-1)}{7}\right]$

f. $3\ln x + 5\ln y - \ln z = \ln\left[\frac{x^3y^5}{z}\right]$

11a. false $\ln e = 1$

b. $\ln x + \ln(2x) = \ln(2x^2)$ false

11c. $\log_6\left(\frac{x-1}{x^2+4}\right) = \log_6(x-1) - \log_6(x^2+4)$ true

d. $\log_3 7 = x \qquad 3^x = 7$ true

$\log_7 3^x = 1$

$x \log_7 3 = 1 \rightarrow x = \frac{1}{\log_7 3}$

12. a. $3^x = 81 \rightarrow 3^x = 3^4 \rightarrow \boxed{x=4}$

b. $4^{2x-1} = 64 \rightarrow 4^{2x-1} = 4^3 \rightarrow 2x-1=3 \rightarrow 2x=4$

$\boxed{x=2}$

c. $3^{1-x} = \frac{1}{27} \rightarrow 3^{1-x} = 3^{-3} \rightarrow 1-x=-3 \rightarrow \boxed{4=x}$

d. $8^{1-x} = 4^{x+2} \rightarrow 2^{3(1-x)} = 2^{2(x+2)} \rightarrow 3-3x = 4x+4$

$\boxed{-1 = x}$

e. $125^x = 625 \rightarrow 5^{3x} = 5^4 \rightarrow 3x=4 \rightarrow \boxed{x=\frac{4}{3}}$

f. $e^{x+4} = \frac{1}{e^{2x}} \rightarrow e^{x+4} = e^{-2x} \rightarrow x+4 = -2x \rightarrow 3x=4$

$\boxed{x=\frac{4}{3}}$

13a. $e^{1-5x} = 793 \rightarrow \ln e^{1-5x} = \ln 793 \rightarrow 1-5x = \ln 793$

$\frac{\ln 793 - 1}{-5} = x \rightarrow \boxed{x = -1.1352}$

b. $7^{x+2} = 10{,}476 \rightarrow \ln 7^{x+2} = \ln(10{,}476) \rightarrow (x+2)\ln 7 = \ln(10{,}476)$

$x = \frac{\ln(10{,}476)}{\ln 7} - 2 \rightarrow \boxed{x = 2.7571}$

c. $\log_4(3x+2) = 3$

$3x+2 = 64 \rightarrow 3x = 62 \rightarrow \boxed{x = \frac{62}{3}}$

d. $\log_2(4x+1) = 5 \rightarrow 4x+1 = 32 \rightarrow 4x = 31 \rightarrow \boxed{x=\frac{31}{4}}$

13e. $5^{2x+3} = 3^{x-1} \to \ln 5^{2x+3} = \ln 3^{x-1} \to$

$(2x+3)\ln 5 = (x-1)\ln 3 \to (2\ln 5)x + 3\ln 5 = (\ln 3)x - \ln 3$

$(2\ln 5 - \ln 3)x = -\ln 3 - 3\ln 5 \to x = \dfrac{-\ln 3 - 3\ln 5}{2\ln 5 - \ln 3}$

$\boxed{x = -2.7954}$

f. $3^{2x} + 3^x - 2 = 0 \qquad u = 3^x \to u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2,\ u = 1$

$3^x = -2$ no solution

$3^x = 1 \to 3^x = 3^0 \to \boxed{x = 0}$

g. $\log_4(x+2) - \log_4(x-1) = 1$

$\log_4\left(\dfrac{x+2}{x-1}\right) = 1 \to \dfrac{x+2}{x-1} = 4 \to x+2 = 4x-4$

$-3x = -6$

$\boxed{x = 2}$

h. $\ln(x-4) + \ln(x+1) = \ln(x+8)$

$\cancel{\ln}[(x-4)(x+1)] = \cancel{\ln}(x+8)$

$x^2 - 3x - 4 = x + 8 \to x^2 - 4x - 12 = 0$

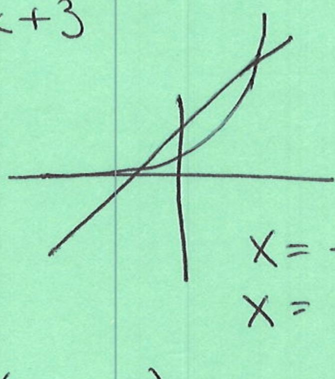
$(x-6)(x+2) = 0$

$x = 6,\ \cancel{x = -2}$

$\ln(-6)$ not defined

$\ln(-1)$ not defined

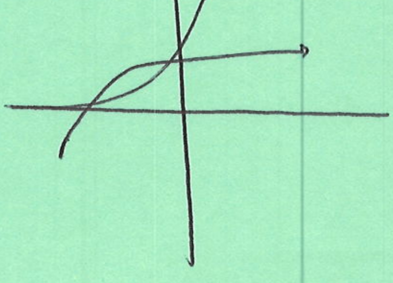
14. a. $3^x = 2x + 3$



$x = -1.3916$

$x = 1.6856$

b. $5^x = \log_3(3x+4)$



$x = -.9023$

$x = .2057$

15. $\frac{1}{2} = e^{k(10)}$

$\ln \frac{1}{2} = k(10) \rightarrow k = \frac{\ln(\frac{1}{2})}{10} = -.0693147$

$A = 16e^{-.0693147x}$

$= 16e^{-.0693147(30)} = 2$

16. $\frac{1}{2} = e^{k(7340)} \rightarrow \frac{\ln(\frac{1}{2})}{7340} = k \quad -9.4434 \times 10^{-5}$

$20\% = A = e^{-9.4434 \times 10^{-5}(x)}$

$\frac{\ln(.2)}{-9.4434 \times 10^{-5}} = x \rightarrow x = 17{,}043$ years

17. $\frac{1}{2} = e^{k(36)} \rightarrow \frac{\ln(\frac{1}{2})}{36} = k = -.019254$

$.90 = e^{-.019254(x)}$

$\frac{\ln(.90)}{-.019254} = x = 5.472$ hours