

Instructions: Show all work. Use exact answers unless specifically asked to round. Answer all parts of each question.

1. Prove the identities.

a. $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

$$\cos\left(\frac{3\pi}{2}\right)\cos x + \sin\left(\frac{3\pi}{2}\right)\sin x = -\sin x$$

b. $1 - \tan^2 x = \frac{\cos 2x}{\cos^2 x}$

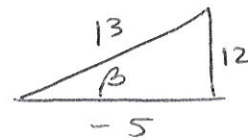
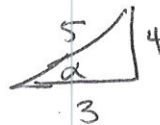
$$1 - \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos 2x}{\cos^2 x}$$

c. $\frac{\sin x - \sin y}{\cos x - \cos y} = -\cot\left(\frac{x+y}{2}\right) = \frac{\cancel{2} \cos\left(\frac{x+y}{2}\right) \cancel{\sin}\left(\frac{x-y}{2}\right)}{-\cancel{2} \cancel{\sin}\left(\frac{x+y}{2}\right) \cancel{\sin}\left(\frac{x-y}{2}\right)} = -\frac{\cos\left(\frac{x+y}{2}\right)}{\sin\left(\frac{x+y}{2}\right)}$

$$= -\cot\left(\frac{x+y}{2}\right)$$

2. Simplify and/or evaluate.

a. $\cos\left[\tan^{-1}\left(\frac{4}{3}\right) + \cos^{-1}\left(-\frac{5}{13}\right)\right]$



$$\left(\frac{3}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) = \frac{-15}{65} - \frac{48}{65} = \frac{-63}{65}$$

b. $\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} = \tan\left(\frac{\pi}{4}\right) = 1$

3. Rewrite $\sin 8x \sin 4x$ as a sum or difference.

$$\frac{1}{2} [\cos(4x) - \cos(12x)] = \frac{1}{2} \cos 4x - \frac{1}{2} \cos 12x$$

Some useful formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\sin\left(\frac{a}{2}\right) = \pm \sqrt{\frac{1 - \cos a}{2}}$$

$$\cos\left(\frac{a}{2}\right) = \pm \sqrt{\frac{1 + \cos a}{2}}$$

$$\tan\left(\frac{a}{2}\right) = \frac{1 - \cos a}{\sin a} = \frac{\sin a}{1 + \cos a}$$

$$\cos(a) \cos(b) = \frac{1}{2} (\cos(a+b) + \cos(a-b))$$

$$\sin(a) \sin(b) = \frac{1}{2} (\cos(a-b) - \cos(a+b))$$

$$\sin(a) \cos(b) = \frac{1}{2} (\sin(a+b) + \sin(a-b))$$

$$\cos(a) \sin(b) = \frac{1}{2} (\sin(a+b) - \sin(a-b))$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= 2 \cos^2 \alpha - 1$$

$$= 1 - 2 \sin^2 \alpha$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\sin(A) + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\sin(A) - \sin(B) = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\cos(A) + \cos(B) = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\cos(A) - \cos(B) = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$