

**Instructions:** Show all work. Use exact answers unless specifically asked to round.

1. Use the method of Separation of Variables to solve the differential equation  $x^2 \frac{dy}{dx} = y - yx$ ,  $y(-1) = -1$ .

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$\frac{dy}{y} = \frac{1-x}{x^2} dx$$

$$\int \frac{dy}{y} = \int \frac{1}{x^2} - \frac{1}{x} dx$$

$$\ln y = \frac{1}{x} - \ln x + C$$

$$y = e^{\frac{1}{x}} \cdot e^{-\ln x} e^C = A e^{\frac{1}{x}} e^{-\ln x} = A e^{\frac{1}{x} - \ln x}$$

$$-1 = A e^{\frac{1}{-1} - \ln(-1)} \rightarrow$$

$$-1 = A e^{-2} \rightarrow A = -e^2$$

$$y = -e^2 e^{\frac{1}{x} - \ln x} = -e^{\frac{1}{x} + 2 - \ln x}$$

2. Describe the procedure for determining if a first order differential equation is homogeneous.

replace  $x$  w/  $tx$  &  $y$  w/  $ty$  if  $t$  factors out & cancels, it's homogeneous

3. Use Euler's Method to estimate the value of the solution at  $t = 1$ ,  $y' = ty + \sqrt{y}$ ,  $y(0) = 1$ , using  $\Delta t = 0.25$ . Carry at least 5 places through your calculation, and round your final answer to 4 decimal places.

$t_n$	$y_n$	$m_n$	$y_{n+1}$
0	1	1	$1(0.25) + 1 = 1.25$
.25	1.25	1.43	$1.43(0.25) + 1.25 = 1.6076$
.5	1.6076	2.0717	$2.07(0.25) + 1.6076 = 2.12557$
.75	2.1256	3.052	$3.052(0.25) + 2.12557 = 2.888596347$
1.0	<u>2.8886</u>		