MTH 265, Quiz #10, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the integrals. Describe or sketch the volume defined by the integral.

   a. $\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\, dxdydz$ $= \int_0^1 \int_0^1 \frac{z\sqrt{1-z^2}}{y+1}\, dy\, dz = \int_0^1 \ln|y+1|\, z\sqrt{1-z^2}\Big|_0^1 dz$

   $(\ln 2 - \cancel{\ln 1}^{\,0}) \int_0^1 z\sqrt{1-z^2}\, dz = -\frac{1}{2}\ln 2\, (1-z^2)^{3/2} \cdot \frac{2}{3} \Big|_0^1 = -\frac{1}{3}\ln 2\left[(0)^{3/2} - (1)^{3/2}\right]$

   $u = 1-z^2 \quad du = -2z \quad \int -\frac{1}{2} u^{1/2}\, du$ $= \frac{\ln 2}{3}$

   b. $\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^3 \cos\phi \sin\phi \cos\theta\, d\rho d\phi d\theta$

   $\int_0^{2\pi} \int_{\pi/2}^{\pi} \frac{1}{4}\rho^4 \Big|_1^2 \cos\varphi \sin\varphi \cos\theta\, d\varphi\, d\theta = \frac{1}{4}(16-1)\, \frac{1}{2}\sin^2\varphi \Big|_{\pi/2}^{\pi} \cdot \int_0^{2\pi} \cos\theta\, d\theta$

   $= \frac{15}{8}(0^2 - 1^2)\, \cancel{\sin\theta}^{\,0} \Big|_0^{2\pi} = 0$

2. Change the integral $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} xz\, dzdxdy$ to cylindrical coordinates and evaluate it. Sketch or describe the region of integration.

$x^2+y^2=4$

$\int_0^{2\pi} \int_0^2 \int_r^2 r\cos\theta\, z\, dz\, r dr d\theta$ $\quad z = \sqrt{x^2+y^2} = r$

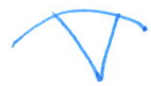
$= \int_0^{2\pi} \int_0^2 \frac{1}{2}z^2 \Big|_r^2 r^2\cos\theta\, dr d\theta = \int_0^{2\pi} \int_0^2 (2 - \frac{1}{2}r^2) r^2 \cos\theta\, dr d\theta =$

$\int_0^{2\pi} \int_0^2 (2r^2 - \frac{1}{2}r^4)\cos\theta\, dr\, d\theta = \int_0^{2\pi} \left(\frac{2}{3}r^3 - \frac{1}{10}r^5\right)\Big|_0^2 \cos\theta\, d\theta =$

$\int_0^{2\pi} \frac{32}{15}\cos\theta\, d\theta = \frac{32}{15}\, \cancel{\sin\theta}^{\,0} \Big|_0^{2\pi} = 0$

3. Change the integral $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} xz\, dzdxdy$ to spherical coordinates and evaluate it. Sketch or describe the region of integration.

   $\rho = \sqrt{8}$ $\quad \varphi = \pi/4$



$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{8}} \rho\cos\theta\sin\varphi\, \rho\cos\varphi\, \rho^2 \sin\varphi\, d\rho\, d\varphi\, d\theta$

$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sqrt{8}} \rho^4 \cos\theta \sin^2\varphi \cos\varphi\, d\rho\, d\varphi\, d\theta = \int_0^{2\pi} \int_0^{\pi/4} \frac{1}{5}\rho^5 \Big|_0^{\sqrt{8}} \cos\theta \sin^2\varphi \cos\varphi\, d\varphi\, d\theta$

$= \frac{64\sqrt{8}}{5} \int_0^{2\pi} \int_0^{\pi/4} \cos\theta \sin^2\varphi \cos\varphi\, d\varphi\, d\theta = \frac{128\sqrt{2}}{5} \cdot \frac{1}{3}\sin^3\varphi \Big|_0^{\pi/4} \int_0^{2\pi} \cos\theta\, d\theta =$

$= \frac{\cancel{128}^{64}\sqrt{2}}{5} \cdot \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^{\cancel{3}\,2} \cdot \cancel{\sin\theta}^{\,0} \Big|_0^{2\pi} = 0$