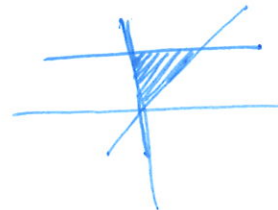


Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the integral $\int_0^1 \int_x^1 e^{x/y} dy dx$ by reversing the order of integration.

$$\begin{aligned} \int_0^1 \int_0^y e^{xy} dx dy &= \int_0^1 y e^{xy} \Big|_0^y dy && \begin{array}{l} y=x \\ y=1 \\ x=0 \end{array} \\ &= \int_0^1 y e^1 dy = \frac{e}{2} y^2 \Big|_0^1 = \frac{e}{2} \end{aligned}$$



2. Set up and evaluate $\iiint_Q x dV$ where Q is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0, y = 3z, z = 0$ in the first octant. Use an appropriate coordinate system.

equivalent to $\iiint_Q z dV$ $y^2 + x^2 = 9, z = 0, y = 3z, x = 0$ first octant

$$\int_0^{\pi/2} \int_0^3 \int_0^{\frac{1}{3}r \cos \theta} z r dz dr d\theta = \int_0^{\pi/2} \int_0^3 \frac{1}{2} z^2 \Big|_0^{\frac{1}{3}r \cos \theta} r dr d\theta =$$

$$\frac{1}{18} \int_0^{\pi/2} \int_0^3 r^3 \cos^2 \theta dr d\theta = \frac{1}{18} \cdot \frac{1}{4} r^4 \Big|_0^3 \int_0^{\pi/2} \frac{1}{2} (1 + \cos 2\theta) d\theta =$$

$$\frac{81}{144} \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\pi/2} = \frac{9}{16} \left[\frac{\pi}{2} + 0 - 0 - 0 \right] = \frac{9\pi}{32}$$

3. Set up and evaluate $\iiint_Q x e^{x^2+y^2+z^2} dV$ where Q is the portion of the unit ball $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant. Use an appropriate coordinate system.

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho \cos \theta \sin \varphi e^{\rho^2} \cdot \rho^2 \sin \varphi d\rho d\varphi d\theta =$$

$$\int_0^{\pi/2} \int_0^{\pi/2} \rho^3 e^{\rho^2} \cos \theta \sin^2 \varphi d\rho d\varphi d\theta = \int_0^{\pi/2} \int_0^{\pi/2} \frac{(\rho^2 - 1)}{2} e^{\rho^2} \Big|_0^1 \cos \theta \sin^2 \varphi d\varphi d\theta =$$

$$= \frac{1}{2} \int_0^{\pi/2} \sin^2 \varphi d\varphi \cdot \sin \theta \Big|_0^{\pi/2} = \frac{1}{4} \int_0^{\pi/2} 1 - \cos 2\varphi d\varphi = \frac{1}{4} \left[\varphi - \frac{1}{2} \sin 2\varphi \right]_0^{\pi/2}$$

$$= \frac{1}{4} \left[\frac{\pi}{2} - 0 \right] = \frac{\pi}{8}$$