MTH 265, Quiz #11, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the integral $\int_0^1 \int_x^1 e^{x/y}\,dy\,dx$ by reversing the order of integration.

$$\int_0^1 \int_0^y e^{x/y}\,dx\,dy = \int_0^1 y e^{x/y}\Big|_0^y dy$$

$y=x$
$y=1$
$x=0$

$$= \int_0^1 y e^1\,dy = \frac{e}{2}y^2\Big|_0^1 = \frac{e}{2}$$

2. Set up and evaluate $\iiint_Q x\,dV$ where Q is bounded by the cylinder $y^2+z^2=9$ and the planes $x=0, y=3x, z=0$ in the first octant. Use an appropriate coordinate system.

equivalent to $\iiint_Q z\,dV$ $\quad y^2+x^2=9,\ z=0,\ y=3z,\ x=0$ first octant

$r=3$ $\qquad \frac{1}{3}r\cos\theta = z$

$$\int_0^{\pi/2}\int_0^3\int_0^{\frac{1}{3}r\cos\theta} z\,r\,dz\,dr\,d\theta = \int_0^{\pi/2}\int_0^3 \frac{1}{2}z^2\Big|_0^{\frac{1}{3}r\cos\theta} r\,dr\,d\theta =$$

$$\frac{1}{18}\int_0^{\pi/2}\int_0^3 r^3\cos^2\theta\,dr\,d\theta = \frac{1}{18}\cdot\frac{1}{4}r^4\Big|_0^3 \int_0^{\pi/2}\frac{1}{2}(1+\cos 2\theta)\,d\theta =$$

$$\frac{81}{144}\left[\theta + \frac{1}{2}\sin 2\theta\Big|_0^{\pi/2} = \frac{9}{16}\left[\pi/2 + 0 - 0 - 0\right] = \frac{9\pi}{32}\right.$$

3. Set up and evaluate $\iiint_Q x e^{x^2+y^2+z^2}\,dV$ where Q is the portion of the unit ball $x^2+y^2+z^2 \le 1$ that lies in the first octant. Use an appropriate coordinate system.

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho\cos\theta\sin\varphi\, e^{\rho^2}\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta =$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 \rho^3 e^{\rho^2}\cos\theta\sin^2\varphi\,d\rho\,d\varphi\,d\theta = \int_0^{\pi/2}\int_0^{\pi/2}\frac{(\rho^2-1)}{2}e^{\rho^2}\Big|_0^1 \cos\theta\sin^2\varphi\,d\theta\,d\varphi$$

$$= \frac{1}{2}\int_0^{\pi/2}\sin^2\varphi\,d\varphi\cdot \underbrace{\sin\theta\Big|_0^{\pi/2}}_{=1} = \frac{1}{4}\int_0^{\pi/2} 1-\cos 2\varphi\,d\varphi = \frac{1}{4}\left[\varphi - \frac{1}{2}\sin 2\varphi\Big|_0^{\pi/2}\right.$$

$$= \frac{1}{4}\left[\pi/2 - 0\right] = \frac{\pi}{8}$$