MTH 265, Quiz #14, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the function $x = \sqrt{y^2 + z^2}$. Identify the surface. Convert the surface to parametric surface form $\vec{r}(u, v)$. Find the equation of the tangent plane at $(5,3,4)$.

Cone wrapped around x-axis $x^2 = y^2 + z^2$

$r = x = u$

$\vec{r}(u,v) = u\hat{\imath} + u\cos v\hat{\jmath} + u\sin v\hat{k}$

$u = 5,\ v = \tan^{-1}\left(\frac{4}{3}\right)$

$\cos v = \frac{3}{5}\quad \sin v = \frac{4}{5}$

$\vec{r}_u = 1\hat{\imath} + \cos v\hat{\jmath} + \sin v\hat{k}$

$\vec{r}_v = 0\hat{\imath} + -u\sin v\hat{\jmath} + u\cos v\hat{k}$

$$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & \cos v & \sin v \\ 0 & -u\sin v & u\cos v \end{vmatrix} = (u\cos^2 v + u\sin^2 v)\hat{\imath} - (u\cos v - 0)\hat{\jmath} + (-u\sin v - 0)\hat{k}$$

$= u\hat{\imath} - u\cos v\hat{\jmath} - u\sin v\hat{k}$

$\Rightarrow 5\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$

$5(x-5) - 3(y-3) - 4(z-4) = 0$

2. Find the arc length of the function $\vec{r}(t) = t^2\hat{\imath} + \ln t\,\hat{\jmath} + t\ln t\,\hat{k}$ on the interval $[1, e]$. After setting up the integral, you may evaluate it numerically (in a calculator).

$r' = 2t\hat{\imath} + \frac{1}{t}\hat{\jmath} + (\ln t + 1)\hat{k}$

$\|r'(t)\| = \sqrt{4t^2 + \frac{1}{t^2} + (\ln t + 1)^2}$

$s = \int_1^e \sqrt{4t^2 + \frac{1}{t^2} + (\ln t + 1)^2}\,dt \approx 7.043257$

3. Find the curvature of the function $\vec{r}(t) = t^2\hat{\imath} + \ln t\,\hat{\jmath} + t\ln t\,\hat{k}$ at the point $(e^2, 1, e)$. Then use that to find the radius of curvature.

$t = e$

$r' = 2t\hat{\imath} + \frac{1}{t}\hat{\jmath} + (\ln t + 1)\hat{k}$

$r'' = 2\hat{\imath} - \frac{1}{t^2}\hat{\jmath} + \frac{1}{t}\hat{k}$

$$K = \frac{\sqrt{\frac{(\ln t + 2)^2}{t^4} + 4\ln^2 t + \frac{16}{t^2}}}{\left[\sqrt{4t + \frac{1}{t^2} + (\ln t + 1)^2}\right]^3}$$

$$r' \times r'' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2t & \frac{1}{t} & \ln t + 1 \\ 2 & -\frac{1}{t^2} & \frac{1}{t} \end{vmatrix} = \left(\frac{1}{t^2} + \frac{\ln t + 1}{t^2}\right)\hat{\imath} - (\cancel{2} - 2\ln t - \cancel{2})\hat{\jmath} + \left(-\frac{2}{t} - \frac{2}{t}\right)\hat{k}$$

$\left(\frac{\ln t + 2}{t^2}\right)\hat{\imath} + 2\ln t\hat{\jmath} - \frac{4}{t}\hat{k}$

$K(e) = \frac{\left(\sqrt{4e^4 + 16e^2 + 9}\right)e}{(4e^3 + 4e^2 + 1)^{3/2}} \approx 0.043272$

$R(e) = \frac{(4e^3 + 4e^2 + 1)^{3/2}}{e\sqrt{4e^4 + 16e^2 + 9}} \approx 23.1098$