MTH 265, Quiz #17, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F}(x, y, z) = -y\hat{\imath} + x\hat{\jmath} - 2\hat{k}$ for the boundary of the surface $S: z^2 = x^2 + y^2, 0 \leq z \leq 4$, oriented downward, using Stokes' Theorem.

$z = \sqrt{x^2+y^2}$ $\quad G = \sqrt{x^2+y^2} - z \quad \nabla G = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}}, -1 \right\rangle$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & -2 \end{vmatrix} = (0-0)\hat{\imath} - (0-0)\hat{\jmath} + (1+1)\hat{k} = 2\hat{k}$$

$$(\vec{\nabla} \times \vec{F}) \cdot \nabla G = -2$$

$$\int_0^{2\pi} \int_0^4 -2\, r\, dr\, d\theta = \int_0^{2\pi} -r^2 \Big|_0^4 d\theta = -16 \cdot 2\pi = -32\pi$$

2. Evaluate the flux $\iint_S \vec{F} \cdot d\vec{S}$ for $\vec{F}(x, y, z) = (\cos z + xy^2)\hat{\imath} + xe^{-z}\hat{\jmath} + (\sin y + x^2 z)\hat{k}$, where S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

$\vec{\nabla} \cdot \vec{F} = y^2 + 0 + x^2 = r^2$ $\quad z = r^2$, $4 = r^2 \Rightarrow r = 2$

$$\int_0^{2\pi} \int_0^2 \int_{r^2}^4 r^2 \cdot r\, dz\, dr\, d\theta = \int_0^{2\pi} \int_0^2 \int_{r^2}^4 r^3\, dz\, dr\, d\theta =$$

$$\int_0^{2\pi} \int_0^2 r^3 z \Big|_{r^2}^4 dr\, d\theta = \int_0^{2\pi} \int_0^2 r^3(4 - r^2)\, dr\, d\theta = \int_0^{2\pi} \int_0^2 4r^3 - r^5\, dr\, d\theta$$

$$= \int_0^{2\pi} r^4 - \frac{1}{6} r^6 \Big|_0^2 d\theta = \int_0^{2\pi} 16 - \frac{32}{3}\, d\theta = 2\pi\left(\frac{16}{3}\right) = \frac{32\pi}{3}$$