MTH 265, Quiz #21, Fall 2018 Name KEY

**Instructions**: Show all work. Use exact answers unless otherwise asked to round.

1. Find the Jacobian for the transformation given by $x = uv, y = \frac{u}{v}$.

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} v & u \\ \frac{1}{v} & -\frac{u}{v^2} \end{vmatrix} = -\frac{u}{v} - \frac{u}{v} = -\frac{2u}{v}$$

2. Determine the change of variables needed for the region bounded by $y = 2x - 1,\ y = 2x + 1,$ $y = 1 - x, y = 3 - x$. Sketch the region in the plane before $(xy)$ and after $(uv)$.

$y - 2x = -1$

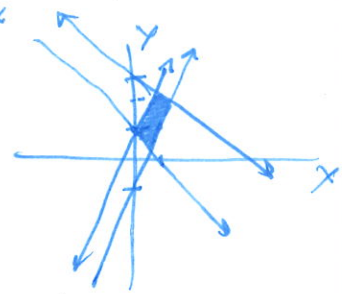
$y - 2x = 1$ $\quad u = y - 2x$

$y + x = 1$

$y + x = 3$ $\quad v = y + x$

$u - v = -3x \Rightarrow -\frac{1}{3}(u - v) = x$

$u + 2v = 3y \Rightarrow \frac{1}{3}(u + 2v) = y$





3. Evaluate the integral $\iint_R xy\,dA$ over the region $R$ bounded by the curves $y = x, y = 3x$, $xy = 1, xy = 3$ using the transformations $x = \sqrt{\frac{v}{u}}, y = \sqrt{vu}$. Sketch the region before the transformation.

$\frac{y}{x} = 1 \quad \frac{y}{x} = 3 \quad u = \frac{y}{x} \quad ux = y$

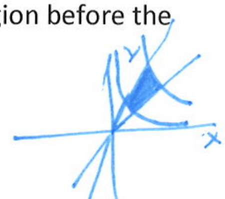
$xy = 1 \quad xy = 3 \quad v = xy \quad v = x^2 u$

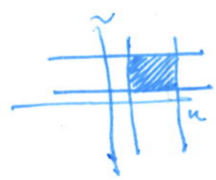
$\frac{v}{u} = x^2 \Rightarrow x = \sqrt{\frac{v}{u}}$

$y = \sqrt{\frac{v}{u}} \cdot u = \sqrt{uv}$

$$J = \begin{vmatrix} -\frac{1}{2} v^{1/2} u^{-3/2} & \frac{1}{2}\frac{1}{\sqrt{uv}} \\ \frac{1}{2}\sqrt{\frac{v}{u}} & \frac{1}{2}\sqrt{\frac{u}{v}} \end{vmatrix} =$$

$-\frac{1}{4u} - \frac{1}{4u} = -\frac{1}{2u}$




$$\int_1^3 \int_1^3 \sqrt{\frac{v}{u}} \cdot \sqrt{uv} \left(-\frac{1}{2u}\right) du\,dv = \int_1^3 \int_1^3 \frac{v}{2u}\,du\,dv =$$

$$\frac{1}{2}\int_1^3 (\ln 3) v\,dv = \frac{\ln 3}{4}(9 - 1) = 2\ln 3$$

4. A ball is thrown eastward into the air from the origin (positive x-axis). The initial velocity is $v_0$ $\langle 50,0,80 \rangle$, with speed measured in feet per second. The spin of the ball results in a southward acceleration of 4 ft/sec$^2$, so the acceleration vector is $\vec{a} = \langle 0, -4, -32 \rangle$. Where does the ball land, and with what speed?

$v = \int \langle 0, -4, -32 \rangle dt = \langle C_1, -4t + C_2, -32t + C_3 \rangle = \langle 50, -4t, -32t + 80 \rangle$

$s = \int \langle 50, -4t, -32t + 80 \rangle dt = \langle 50t + C_1, -2t^2 + C_2, -16t^2 + 80t + C_3 \rangle =$

$\langle 50t, -2t^2, -16t^2 + 80t \rangle$

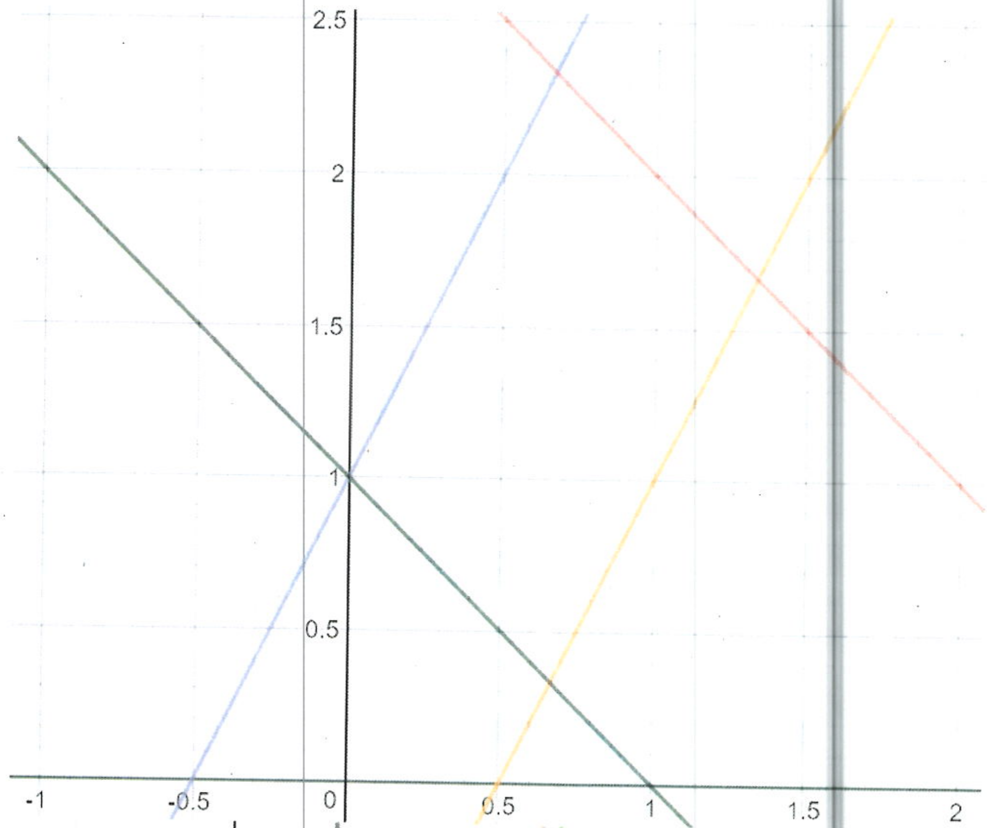
ground $z = 0$ $\quad -16t^2 + 80t = 0$

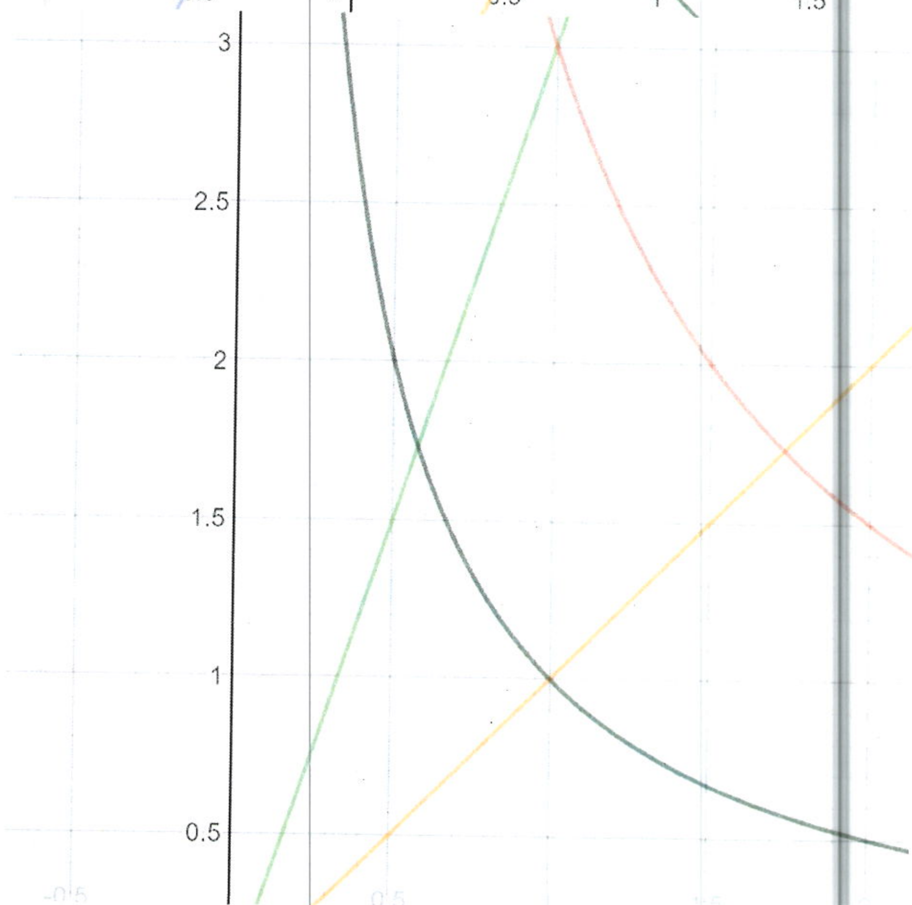
$t = 0 \quad -16t + 80 = 0$

$t = 5$

$s(5) = \langle 250, -50, 0 \rangle$

$v(5) = \langle 50, -20, -80 \rangle \quad \|v(5)\| = \sqrt{9300} \approx 96.44$


2.5
2
1.5
1
0.5
-1
-0.5
0
0.5
1
1.5
2


3
2.5
2
1.5
1
0.5