MTH 265, Quiz #2, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the domain and range of $f(x,y) = \sqrt{9 - x^2 - 9y^2}$ and sketch the domain in the plane.

$9 - x^2 - 9y^2 \geq 0$

$9 \geq x^2 + 9y^2$

$\frac{x^2}{9} + y^2 \leq 1$

domain: $\{(x,y) \mid \frac{x^2}{9} + y^2 \leq 1\}$ Range $0 \leq z \leq 3$

inside or on the ellipse

largest possible value of $9 - x^2 - 9y^2$ is at $(0,0) \Rightarrow 9 \Rightarrow \sqrt{9} = 3$

smallest possible value of $9 - x^2 - 9y^2$ when on ellipse $\Rightarrow 0$

2. Consider the function $f(x,y) = \sqrt{x^2 - y^2}$. Convert the function to the indicated coordinate system or format.
   a. Write f in spherical coordinates.

$z = \sqrt{x^2 - y^2} \Rightarrow z^2 = x^2 - y^2$

$\rho^2 \cos^2\varphi = \rho^2 \sin^2\varphi \cos^2\theta - \rho^2 \sin^2\varphi \sin^2\theta$

$\cos^2\varphi = \sin^2\varphi(\cos^2\theta - \sin^2\theta)$

$\cot^2\varphi = \cos^2\theta - \sin^2\theta = \cos 2\theta$ or $\cot\varphi = \sqrt{\cos 2\theta}$

   b. Write f in cylindrical coordinates.

$z = \sqrt{r^2\cos^2\theta - r^2\sin^2\theta} = r\sqrt{\cos^2\theta - \sin^2\theta}$

$= r\sqrt{\cos 2\theta}$

   c. Write f in parametric surface form, $\vec{r}(u,v)$.

(1) $r(u,v) = u\hat{\imath} + v\hat{\jmath} + \sqrt{u^2 - v^2}\,\hat{k}$

$z^2 + y^2 = x^2$
like a circle
$u = x \to$ like radius in y, z

(2) $r(u,v) = u\hat{\imath} + u\cos v\,\hat{\jmath} + u\sin v\,\hat{k}$