MTH 265, Quiz #8, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Evaluate the integral $\int_1^3 \int_1^5 \frac{\ln y}{xy}\,dy dx$. $= \int_1^3 \frac{1}{x}\left[\int_1^5 \frac{\ln y}{y}\,dy\right]dx$ $u = \ln y$, $du = \frac{1}{y}dy$

$\int u\,du = \frac{1}{2}u^2 = \frac{1}{2}(\ln y)^2$

$$\int_1^3 \frac{1}{x}\left[\frac{1}{2}\ln^2 y\,\Big|_1^5 dx = \frac{1}{2}\int_1^3 \frac{1}{x}\left(\ln^2 5 - \cancel{\ln^2 1}^{\,0}\right)dx = \frac{\ln^2 5}{2}\int_1^3 \frac{1}{x}\,dx$$

$$= \frac{\ln^2 5}{2}\ln x\,\Big|_1^3 = \frac{\ln^2 5}{2}\cdot\left(\ln 3 - \cancel{\ln 1}^{\,0}\right) = \frac{\ln^2 5\cdot \ln 3}{2}$$

2. Evaluate $\int\int_R (x^2+2y)dA$ over the region R bounded by $y = x, y = x^3, x \geq 0$.

$$\int_0^1 \int_{x^3}^{x} x^2+2y\,dy dx =$$

$x = x^3$ when $x = -1, 0, 1$

$$\int_0^1 x^2y + y^2\Big|_{x^3}^{x} dx = \int_0^1 x^3 + x^2 - x^5 - x^6\,dx$$

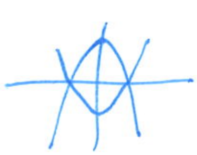
$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{6}x^6 - \frac{1}{7}x^7\Big|_0^1 = \frac{1}{4} + \frac{1}{3} - \frac{1}{6} - \frac{1}{7} = \frac{23}{84}$$

3. Find the volume of the solid bounded by $y = 1 - x^2,\ y = x^2 - 1,\ x + y + z = 2,$
$2x + 2y - z = 10$. Write a double integral and then evaluate it. Sketch the region in the plane.

$2x+2y-10 = z$

$2-x-y = z$

$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2}\int_{2x+2y-10}^{2-x-y} dz\,dy\,dx =$$



$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2} 2-x-y-(2x-2y-10)\,dy\,dx =$$

$$\int_{-1}^{1}\int_{x^2-1}^{1-x^2} 12-3x-3y\,dy\,dx = \int_{-1}^{1} 12y-3xy-\frac{3}{2}y^2\Big|_{x^2-1}^{1-x^2} dx$$

$$= \int_{-1}^{1} 6x^3-2x^2-6x+24\,dx = \frac{6}{4}x^4-\frac{2}{3}x^3-3x^2+24x\Big|_{-1}^{1} = 32$$