MTH 266, Homework #2, Fall 2018   Name ______________________________

Instructions: Write your work up neatly and attach to this page. Use exact values unless specifically asked to round. Show all work.

1. Consider the following matrices.

$$A=\begin{bmatrix}3&1\\-1&4\end{bmatrix}, B=\begin{bmatrix}9&3\\1&0\end{bmatrix}, C=\begin{bmatrix}2&-2\\-4&4\end{bmatrix}, D=\begin{bmatrix}1&3&4\\-2&1&0\\3&-4&1\end{bmatrix}, E=\begin{bmatrix}0&-3&5\\1&-4&0\\-1&2&-7\end{bmatrix}$$

$$F=\begin{bmatrix}1&3&-2&0\\2&4&-1&5\end{bmatrix}, G=\begin{bmatrix}6&-7\\11&-5\\2&3\end{bmatrix}, H=\begin{bmatrix}1&0&-2\end{bmatrix}, J=\begin{bmatrix}4\\-1\end{bmatrix}$$

a. Add
   - ii. A+B
   - ii. 2B-3C

b. Multiply
   - i. 4A
   - ii. -5H

c. Multiply. If the multiplication is not possible, explain why not.
   - i. AB
   - ii. DE
   - iii. GC
   - iv. EH
   - v. BA
   - vi. BF
   - vii. DG
   - viii. BJ

d. Find the transpose of
   - i. A
   - ii. G
   - iii. D
   - iv. J

e. Find the inverse of the matrix if it exists.
   - i. A
   - ii. D
   - iii. C
   - iv. F.

f. Find F^TF and GG^T. Are the matrices symmetric?

2. If $f(x) = x^3 - 2x^2 + 5x - 10$, find $f(A)$ if $A = \begin{bmatrix}2&1&-1\\1&0&2\\-1&1&3\end{bmatrix}$. Why can we only calculate matrix polynomials for square matrices? [Note: Treat 10 like $10I$ where I is the appropriate identity.]

3. Evaluate the expression $A^2 - 4A - 5I$ for the matrix $A = \begin{bmatrix}1&2\\4&3\end{bmatrix}$.

4. If $A = \begin{bmatrix}-2&-1\\1&0\\3&-4\end{bmatrix}, B = \begin{bmatrix}0&3\\2&0\\-4&-1\end{bmatrix}$, solve for X in $2X + 3A = B$.

5. Write a 3x3 matrix which has the effect of the row operations described.
   a. Multiply the second row by -5.
   c. Switch rows 1 and 2.
   b. Add the third row to the first row.
   d. Multiply row two by 3 and add that row to row three.

6. For each statement below determine if it is true or false. If the statement is false, briefly explain why it is false and give the true statement.
   a. $(ABC)^T = C^T B^T A^T$

   b. If A is invertible, then elementary row operations that reduce A to the identity I_n also reduce A^{-1} to I_n.

   c. If A and B are invertible, then $A^{-1}B^{-1}$ is the inverse of AB.

   d. If A can be reduced to the identity matrix, then A must be invertible.

   e. If A^T is not invertible, then A is not invertible.

   f. If there is an *nxn* matrix D such that AD=I, then DA=I.

7. For each of the matrices below, determine if they are invertible. Use as few calculations as necessary.

   a. $\begin{bmatrix} -4 & 2 \\ 6 & -3 \end{bmatrix}$

   b. $\begin{bmatrix} -5 & 1 & 4 \\ 0 & 0 & 0 \\ 1 & 4 & 9 \end{bmatrix}$

   c. $\begin{bmatrix} 3 & 4 & 7 & 4 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   d. $\begin{bmatrix} 5 & 3 & 1 & 7 & 9 \\ 6 & 4 & 2 & 8 & -8 \\ 7 & 5 & 3 & 10 & 9 \\ 9 & 6 & 4 & -9 & -5 \\ 8 & 5 & 2 & 11 & 4 \end{bmatrix}$

8. For each of the systems of equations below, write the system in matrix form, and then write the solution in matrix form using inverses. Find the inverse and compute the solution. For the 3x3 inverses, you may use your calculator.

   a. $\begin{cases} 2x + 3y = 12 \\ 4x - y = 10 \end{cases}$

   c. $\begin{cases} -x + 5y = 17 \\ 3x - 4y = 12 \end{cases}$

   b. $\begin{cases} 5x - y + 2z = 10 \\ 3x + 2y - 4z = 16 \\ -4x - 3y + z = 7 \end{cases}$

   d. $\begin{cases} x + y + z = 9 \\ -x + 2y - 3z = 14 \\ 3x - 5y - 2z = -18 \end{cases}$

9. The matrix $P = \begin{bmatrix} .6 & .1 & .1 \\ .2 & .7 & .1 \\ .2 & .2 & .8 \end{bmatrix}$ represents the proportion of the voting population that changes from one party to another between election cycles. The columns represent Republican, Democratic and Independent parties respectively. Find and interpret the meaning of P^2.

10. Use a rectangular coordinate system to plot $\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \vec{v} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and their images under the given transformation T. Describe geometrically what T does to each vector in R^2. Plot each on a separate graph. Label each transformation as one-to-one, onto, both or neither.

a. $T(\vec{x}) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

b. $T(\vec{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c. $T(\vec{x}) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \theta = \frac{5\pi}{4}$

d. $T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

e. $T(\vec{x}) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \theta = \frac{\pi}{3}$