MTH 266, Homework #3, Fall 2018 Name ____

Instructions: Write your work up neatly and attach to this page. Use exact values unless specifically asked to round. Show all work.

1. Calculate the determinants of the following matrices using cofactor expansion (except for a). Try to use the expansion with the least number of steps.

   a. $\begin{bmatrix} -1 & 3 \\ 4 & 2 \end{bmatrix}$

   b. $\begin{bmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{bmatrix}$

   c. $\begin{bmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   d. $\begin{bmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{bmatrix}$

   e. $\begin{bmatrix} 6 & 3 & 2 & 4 & 0 \\ 9 & 0 & -4 & 1 & 0 \\ 8 & -5 & 6 & 7 & 1 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 2 & 3 & 2 & 0 \end{bmatrix}$

   f. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Use row operations to reduce the matrix to echelon form and then find the determinant of the original matrix.

   a. $\begin{bmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{bmatrix}$

   b. $\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & -1 & -3 & 0 \\ 0 & 1 & 5 & 4 \\ -1 & 2 & 8 & 5 \\ 3 & -1 & -2 & 3 \end{bmatrix}$

   d. $\begin{bmatrix} -3 & -2 & 1 & -4 \\ 1 & 3 & 0 & -3 \\ -3 & 4 & -2 & 8 \\ 3 & -4 & 0 & 4 \end{bmatrix}$

3. Find the determinants of the matrices below using the fact that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$.

   a. $\begin{bmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{bmatrix}$

   b. $\begin{bmatrix} a & b & c \\ g & h & i \\ 2d+a & 2e+b & 2f+c \end{bmatrix}$

4. Given your answers in problems #1 and #2, which matrices are invertible and which are not.

5. Using theorems of determinants only, prove or find the following.

   a. Show that if A is invertible then $\det A^{-1} = \dfrac{1}{\det A}$.

   b. Let A & B be square matrices. Show that $det(AB) = det(BA)$.

   c. Let A and P be square matrices with P invertible. Show that $det(PAP^{-1}) = det(A)$.

   d. Suppose A is a square matrix such that $det(A^4) = 0$. Show that A is not invertible.

6. If $det(A) = (-1)$ and $det(B) = 2$, which are both $n \times n$ matrices, find

   i. $det(AB)$
   ii. $det(B^5)$
   iii. $det(2A)$
   iv. $det(A^T A)$
   v. $det(B^{-1}AB)$

7. Find the area for the parallelogram bounded by:

   a. The vectors $\vec{u} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \vec{v} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$.
   b. The points $(1,0), (6,2), (4,4), (-1,2)$.

8. Find the volume of the parallelepiped bounded by the vertices $\vec{u} = \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 8 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} 6 \\ 1 \\ -1 \end{bmatrix}$.

9. Find the volume of the tetrahedron with the vertices $(3,-1,1), (4,-4,4), (1,1,1), (0,0,1)$.

10. For each statement below determine if it is true or false. If the statement is false, briefly explain why it is false and give the true statement.

    a. A row replacement does not affect the determinant of a matrix.

    b. If two row interchanges are made in succession, then the new determinant is the same as the original.

    c. The determinant of A is the product of the diagonal entries of A.

    d. If $det\ A$ is zero, then two rows or two columns of A are the same, or a row or a column is zero.

    e. $det\ A^T = (-1) det\ A$.