MTH 266, Homework #7, Fall 2018 Name ______________________________

Instructions: Write your work up neatly and attach to this page. Use exact values unless specifically asked to round. Show all work.

1. Find the characteristic polynomial and all eigenvalues and eigenvectors for each matrix. Use those complex eigenvalues to create a matrix P and C so that P is a similarity transformation and C is similar to the original matrix. If the eigenvalues are real, find the similarity transformation P that diagonalizes the matrix, and D the diagonal matrix.
   a. $\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$
   b. $\begin{bmatrix} -3 & -8 \\ 4 & 5 \end{bmatrix}$
   c. $\begin{bmatrix} -4 & 1 \\ 6 & -5 \end{bmatrix}$
   d. $\begin{bmatrix} -4 & 5 \\ -5 & -4 \end{bmatrix}$
   e. $\begin{bmatrix} -2 & 5 & 3 \\ 0 & 2 & -4 \\ 0 & -1 & 2 \end{bmatrix}$
   f. $\begin{bmatrix} 0 & 5 \\ -2 & 2 \end{bmatrix}$
   g. $\begin{bmatrix} -3 & 7 \\ 5 & -1 \end{bmatrix}$
   h. $\begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$
   i. $\begin{bmatrix} -2 & 2 \\ -5 & 6 \end{bmatrix}$
   j. $\begin{bmatrix} 4 & -2 & 0 \\ 0 & 0 & -3 \\ 0 & 2 & 5 \end{bmatrix}$

2. Diagonalize each matrix, if possible. Find the eigenvalues, if none are given, and state the eigenspace for each. For each matrix that is diagonalizable, find the value of A^k.
   a. $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$
   b. $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$
   c. $\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$, λ=1,5
   d. $\begin{bmatrix} 2 & -2 & -2 \\ 3 & -3 & -2 \\ 2 & -2 & -2 \end{bmatrix}$, λ=-2,-1,0

3. A is a 4x4 matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. Is it possible that A is *not* diagonalizable? Justify your answer.

4. Define $T: R^2 \to R^2$ by $T(\vec{x}) = A\vec{x}$. Find a basis $\mathscr{B}$ for R^2 with the property that $[T]_B$ is diagonal, given $A = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$.

5. A 2x2 matrix is known to have the eigenvalues -3 and 4. What are the eigenvalues of the following matrices?
   a. $2A$ b. $5A$ c. $A - 3I$ d. $A + 4I$ e. $2A + 5I$

6. For each statement, indicate whether it's true or false. For the ones that are false, state the correct true statement.
   a. If $A\vec{x} = \lambda\vec{x}$ for some vector $\vec{x}$, then λ is an eigenvalue of A.
   b. A matrix is not invertible if and only if 0 is an eigenvalue of A.

c. If $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

d. The eigenvalues of a matrix are on its main diagonal.

e. An nxn matrix can have more than n eigenvalues.

f. The determinant of A is the product of the diagonal entries of A.

g. The elementary row operations of A do not change its determinant.

h. If λ+5 is a factor of the characteristic polynomial, then 5 is an eigenvalue of A.

i. If R^n has a basis of eigenvectors of A, then A is diagonalizable.

j. If A is diagonalizable, then A is invertible.

7. For each of the problems below, find the general solution to the system of linear ODEs, and plot a few trajectories of the system. Be sure to use arrows on the trajectories and eigenvectors to indicate the direction of motion.

a. $\vec{x}' = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \vec{x}$

b. $\vec{x}' = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \vec{x}$

c. $\vec{x}' = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} \vec{x}$

d. $\vec{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \vec{x}$

e. $\vec{x}' = \begin{bmatrix} 4 & -3 \\ 6 & -2 \end{bmatrix} \vec{x}$