MTH 266, Quiz #3, Fall 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Rewrite the following system of equations in a) an augmented matrix form, b) vector equation form, c) matrix equation form. Are the vectors in the A matrix (coefficient matrix for the system) linearly independent? Why or why not?

$$\begin{cases} 5x_1 - x_2 + 2x_3 = 7 \\ -2x_1 + 6x_2 + 9x_3 = 0 \\ -7x_1 + 5x_2 - 3x_3 = -7 \end{cases}$$

reduces to $\begin{bmatrix} 1 & 0 & 0 & | & 24/13 \\ 0 & 1 & 0 & | & 10/13 \\ 0 & 0 & 1 & | & -2/13 \end{bmatrix}$

a) $\left[\begin{array}{ccc|c} 5 & -1 & 2 & 7 \\ -2 & 6 & 9 & 0 \\ -7 & 5 & -3 & -7 \end{array}\right]$

b) $\begin{bmatrix} 5 \\ -2 \\ -7 \end{bmatrix} x_1 + \begin{bmatrix} -1 \\ 6 \\ 5 \end{bmatrix} x_2 + \begin{bmatrix} 2 \\ 9 \\ -3 \end{bmatrix} x_3 = \begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$

c) $\begin{bmatrix} 5 & -1 & 2 \\ -2 & 6 & 9 \\ -7 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix}$

2. Row reduce the system $A\vec{x} = \vec{b}$ given by $\begin{cases} 2x_1 + 2x_2 + 4x_3 = 8 \\ -4x_1 - 7x_2 - 11x_3 = -4 \\ \quad -3x_2 - 3x_3 = 12 \end{cases}$. State the solution to the system in parametric form (if needed). Is the matrix A in this question invertible? If it is, find its inverse. If it is not, explain why not.

reduces to $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

A is not invertible
Solution is dependent
determinant is 0

$x_1 + x_3 = 8 \Rightarrow x_1 = -x_3 + 8$

$x_2 + x_3 = -4 \Rightarrow x_2 = -x_3 - 4$

$x_3 = x_3 \Rightarrow x_3 = x_3$

$\Rightarrow \vec{x} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} x_3 + \begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$