MTH 266, Quiz #4, Fall 2018 Name KEY

**Instructions**: Show all work. Use exact answers unless otherwise asked to round.

1. Consider the transformation $T: P_n \to P_{n+1}$ such that $T(f) = \int_0^t f(x)dx$. If $f(x)$ is any polynomial in $P_n$, use the definition of a linear transformation to show that $T$ is linear.

does $T(f+g) = T(f) + T(g)$? yes

since $\int_0^t (f(x)+g(x))dx = \int_0^t f(x)dx + \int_0^t g(x)dx$ by properties of integrals

does $T(kf) = kT(f)$? yes

since $\int_0^t kf(x)dx = k\int_0^t f(x)dx$ by properties of integrals

does $T(\vec{0}) = \vec{0}$? yes

since $\int_0^t 0\,dx = 0$ in $P_{n+1}$

So $T$ is linear

2. Compare Problem #1 to the following: Consider the transformation $T: R^3 \to R^3$ such that $T(\vec{x}) = A\vec{x}$. If $\vec{x}$ is any vector in $R^3$, use the definition of a linear transformation to show that $T$ is linear. [Hint: use properties of a matrix.]

by using properties of a matrix we can show that

$T(\vec{x}+\vec{y}) = A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y} = T(\vec{x}) + T(\vec{y})$

and

$T(k\vec{x}) = A(k\vec{x}) = k(A\vec{x}) = kT(\vec{x})$

and

$T(\vec{0}) = A(\vec{0}) = \vec{0}$

Therefore, any transformation which can be written as a matrix must be linear.