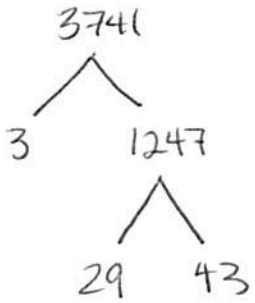
MT 118, Exam #2, Fall 2019 Name KEY

Instructions: Show all work. Partial credit can only be given where work is shown. Be sure to answer all parts of each question. You may not use a calculator on this quiz.

1. Find a factor tree for 3741. Write the final result as prime factors. (6 points)



$3^1 \cdot 29^1 \cdot 43^1$

2. Consider the statement: $6|xy$ then $6|x$ or $6|y$. Determine if the statement is true or false. If true, explain your reasoning. If false, give a counterexample. (5 points)

False $6|12 = 3 \cdot 4$ but $6 \nmid 3$ and $6 \nmid 4$

This only works when the # replacing 6 is prime

3. Check each of the numbers for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 11. You don't need to factor the number but say which of these numbers divides each. Partial credit is possible only if you show work. (5 points each)

a. 2,199,456

$2+1+9+9+4+5+6 = 36$

$4|56$ $2+9+4+6 = 21$

$8|456$ $1+9+5 = 15$

2, 3, 4, 6, 8, 9

not 5, 10, 11

b. 385,627

$3+8+5+6+2+7 = 31$ not even not 2, 3, 4, 5, 6, 8, 9, 10 not 11

$3+5+2 = 10$

$8+6+7 = 21$

11

c. 325,608

$3+2+5+6+0+8 = 24$ not 5 $8|608$

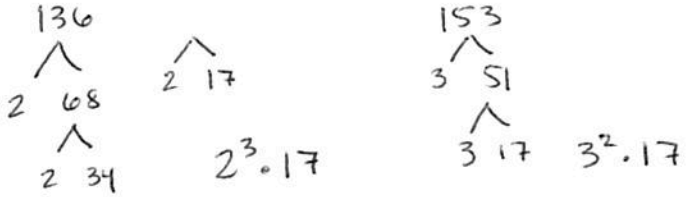
$3+5+0 = 8$ not 9, 10

$2+6+8 = 16$

2, 3, 4, 6, 8,

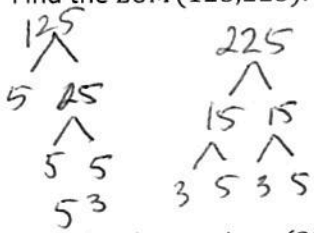
4. Find the $GCF(136,153)$. (4 points)

17



$2^3 \cdot 17$ $3^2 \cdot 17$

5. Find the $LCM(125,225)$. (4 points)



$3^2 \cdot 5^3 = 1125$

~~$3^2 \cdot 5^2$~~

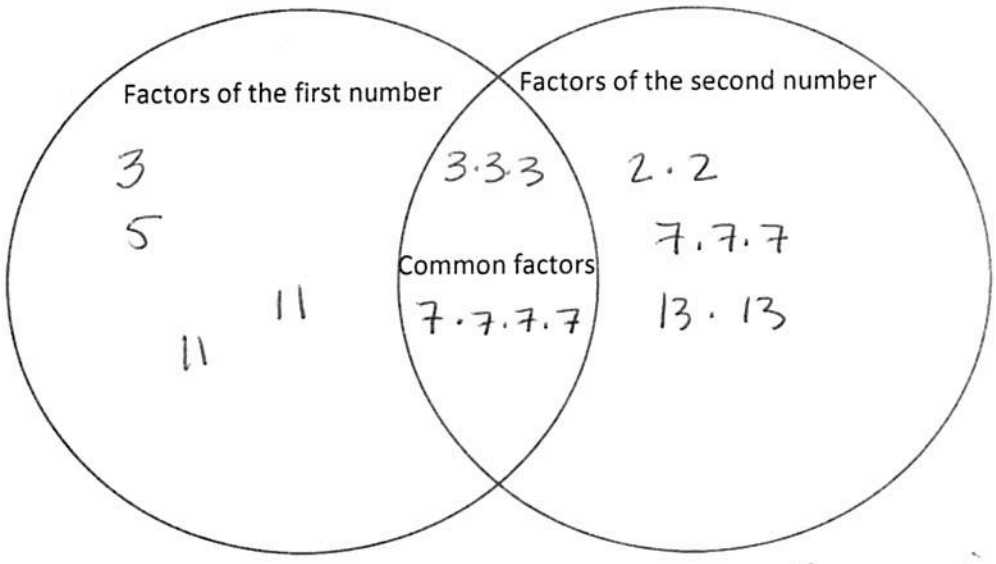
6. Consider the numbers $(3^4 \cdot 5 \cdot 7^4 \cdot 11^2, 2^2 \cdot 3^3 \cdot 7^7 \cdot 13^3)$. (6 points)
   a. Find GCF

$3^3 7^4$

   b. Find LCM

$2^2 \cdot 3^4 \cdot 5 \cdot 7^7 \cdot 11^2 \cdot 13^3$

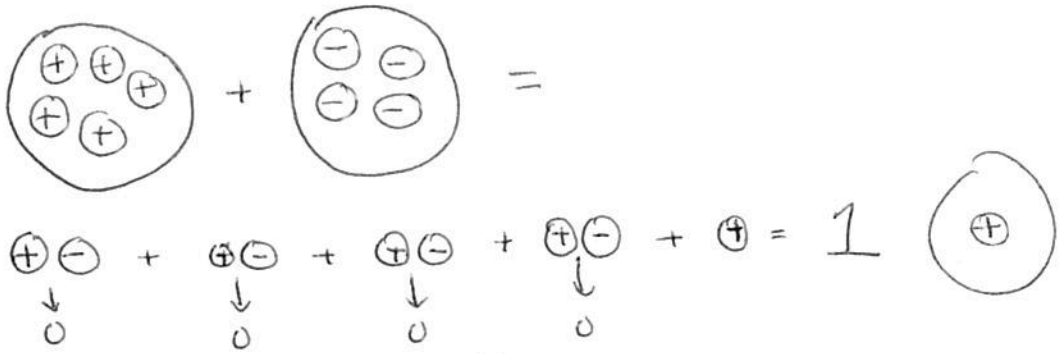
7. Consider the Venn diagram below. Explain how a diagram like this can help someone find the GCF and LCM of two numbers in prime factored form. (Hint: You can use problem 6 as your example.) (5 points)



explanations will vary but LCM is the union of all factors while GCF is the intersection

8. Explain how to use charged particles to illustrate the arithmetic problem $5 + (-4)$. Draw a picture! (5 points)

answers will vary



9. Calculate the following expressions. (3 points each)
   a. $3 - 7$ -4
   b. $8 - (-4)$ 12
   c. $(-7) - (-8)$ 1
   d. $-18 \div 3$ -6
   e. $-(-3)(-5)$ -15
   f. $-3(-7 - 6)$ $= -3(-13) = 39$

10. Order the following numbers from smallest to largest. (4 points)

$\{-5, 3, 8, -11, 0, -108, 15\}$

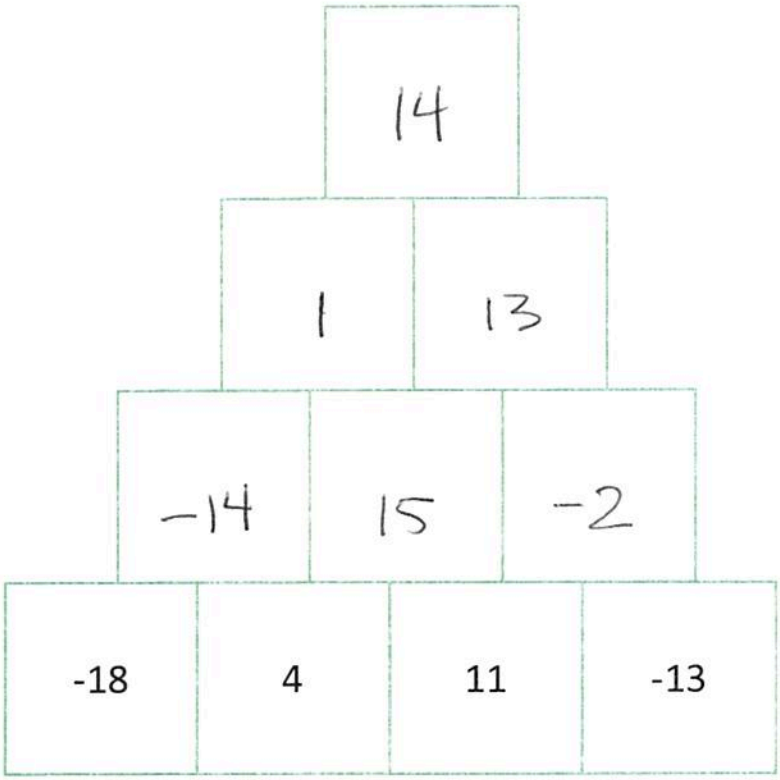
$-108, -11, -5, 0, 3, 8, 15$

11. Are the following numbers positive or negative? You don't need to fully evaluate the expression, just find the sign. (2 points each)
   a. $(-2)^5$ negative
   b. $(-1)^{20}$ positive
   c. $-(-3)^{-4}$ negative

12. Write the number 0.00000003048 in scientific notation. (4 points)

3.048×10^{-8}

13. Fill in each empty square so that the number in the square will be the sum of the pair of numbers beneath the square. (6 points)



14. Perform the indicated operations. You must show work. (5 points each)

a. $\frac{8}{9}+\frac{1}{12}+\frac{3}{16}$

$\frac{16}{16}\cdot\frac{8}{9}+\frac{12}{12}\cdot\frac{1}{12}+\frac{9\cdot 3}{9\ 16}=\frac{128}{144}+\frac{12}{144}+\frac{27}{144}=\frac{167}{144}$

LCD = 144

$1\frac{23}{144}$ or $\frac{167}{144}$

b. $\frac{13}{18}-\frac{8}{27}$

$\frac{3}{3}\cdot\frac{13}{18}-\frac{2}{2}\cdot\frac{8}{27}=\frac{26}{54}-\frac{16}{54}=\frac{10}{54}=\frac{5}{27}$

c. $2\frac{2}{5}+3\frac{5}{8}-3\frac{5}{6}$ $=\frac{12}{5}+\frac{29}{8}-\frac{23}{6}=\frac{24}{24}\cdot\frac{12}{5}+\frac{15}{15}\cdot\frac{29}{8}-\frac{20}{20}\cdot\frac{23}{6}=$

$\frac{288}{120}+\frac{435}{120}-\frac{460}{120}=\frac{263}{120}$

$2\frac{23}{120}$ or $\frac{263}{120}$

d. $\frac{1}{\cancel{4}_{2}} \times \frac{3}{\cancel{8}_{2}} = \frac{12}{56} = \frac{3}{14}$

e. $\frac{12}{13} \div \frac{6}{5} = \frac{\cancel{12}^{2}}{13} \cdot \frac{5}{\cancel{6}} = \frac{10}{13}$

15. For the numbers in the set $\left\{\frac{55}{7}, -11, 4, \sqrt{64}, -6.75, 14000, \sqrt{19}, \pi^2, \frac{0}{3}, 0.\overline{69}, \frac{16}{8}\right\}$, determine which of the numbers belongs to each of the number types below. (4 points each)

a. Natural (Counting) Numbers

$4, 8, 14000, 2$

b. Integers

$-11, 4, 8, 14000, 0, 2$

c. Rational Numbers

$\frac{55}{7}, -11, 4, 8, -6.75, 14000, 0, 0.\overline{69}, 2$

d. Irrational Numbers

$\sqrt{19}, \pi^2$

16. Solve equation or inequality. (4 points each)

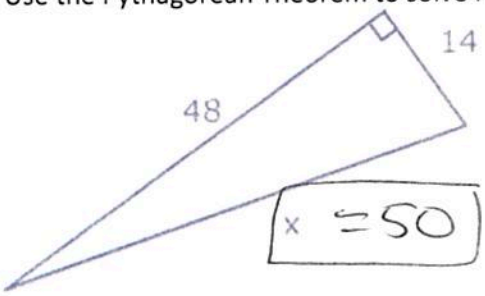
a. $\frac{1}{6}x = -\frac{5}{12}$ $\quad x = -\frac{5}{\cancel{12}_{2}} \cdot \frac{\cancel{6}^{1}}{1} = \boxed{-\frac{5}{2}}$

b. $-\frac{1}{3}x < -\frac{5}{6}$

$x > +\frac{5}{\cancel{6}_{2}}\left(+\frac{\cancel{3}}{1}\right) = \frac{5}{2}$

$\boxed{x > \frac{5}{2}}$

17. Use the Pythagorean Theorem to solve for the missing side. (5 points)



$14^2 + 48^2 = 2500 = 50^2$

18. Rewrite and simplify the expression $16^{3/4}$. (3 points)

$\sqrt[4]{16^3} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

19. Explain the difference between $\sqrt[4]{-16}$ and $-16^{1/4}$. One is real and one is not. Which is the real value? (4 points)

$\sqrt[4]{-16}$ is not real since 4th roots, like square roots cannot apply to negative #'s (all even powers are positive) whereas $-16^{1/4} = -\sqrt[4]{16} = -2$ because negative is outside the root.

20. Simplify $\frac{\sqrt{147}}{\sqrt{12}}$. (3 points)

$\frac{\sqrt{147}}{\sqrt{12}} = \frac{\sqrt{3 \cdot 49}}{\sqrt{3 \cdot 4}} = \frac{\sqrt{3} \cdot \sqrt{49}}{\sqrt{3} \cdot \sqrt{4}} = \boxed{\frac{7}{2}}$

21. Which number is larger, $\pi^{\sqrt{2}}$ or $(\sqrt{2})^{\pi}$? Justify your answer. (3 points)

5.047497267 $\quad$ 2.9706864424

$\pi^{\sqrt{2}} > (\sqrt{2})^{\pi}$

22. Is the set of irrational numbers closed under addition? Why or why not? (6 points)

no. $1+\sqrt{2}$ is irrational and $1-\sqrt{2}$ is irrational, but $(1+\sqrt{2}) + (1-\sqrt{2}) = 2$ which is not irrational

examples will vary.

23. Calculate $(4 \otimes 3) \ominus (4 \otimes 5)$ in the 6-clock. (5 points)

$\equiv 12 - 20 \equiv$

$\equiv -8 + 6 \equiv -2 + 6 \equiv$ **4 mod 6**

24. List three numbers that are equivalent *mod* 7 to -2. (3 points)

$-2 - 7 = -9$

$-2 + 7 = 5$

$-2 + 7(2) = 12$

$-2 + 7(3) = 19$

any # that has a remainder of 5 when divided by 7

Bonus: What is the smallest counting number divisible by (all) 2, 4, 6, 8, 10, 12, 14. (10 points)

i.e. LCM

$2 \quad 2^2 \quad 2 \cdot 3 \quad 2^3 \quad 2 \cdot 5 \cdot 2^2 \cdot 3 \cdot 2 \cdot 7$

$2^3 \cdot 3 \cdot 5 \cdot 7 = 840$