EAS 596, Fall 2019, Homework 2
Due Monday 9/16, **4:00 PM**, Box outside Jarvis 326 or in class

Work all problems. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, that work will not be graded. All electronic work (m-files, etc.) **must** be submitted through UBLearns by the due time shown above. Electronic files must obey the following naming convention: `ubitname_hw2_p1.m`, replacing ubitname with your ubitname. Any handwritten work may be submitted in class. Each problem will be graded according to the following scheme:

- 2 Points: Solution is complete and correct,
- 1 Points: Solution is incorrect or incomplete but was using the correct ideas,
- 0 Points: Using incorrect ideas.

1. In the xy plane, mark all nine of these linear combinations:

$$c\begin{bmatrix}2\\1\end{bmatrix}+d\begin{bmatrix}0\\1\end{bmatrix} \quad \text{with } c=0,1,2 \text{ and } d=0,1,2$$

2. (a) Find vector $\boldsymbol{v}$ and $\boldsymbol{w}$ so that $\boldsymbol{v}+\boldsymbol{w}=(4,5,6)$ and $\boldsymbol{v}-\boldsymbol{w}=(2,5,8)$.
   (b) This is a question with ______ unknown numbers and an equal number of equations to find those numbers.

3. (a) Find unit vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ in the directions of $\boldsymbol{v}=(3,1)$ and $\boldsymbol{w}=(2,1,2)$.
   (b) Find unit vectors $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ that are perpendicular to $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$.

4. (a) If $\|\boldsymbol{v}\|=5$ and $\|\boldsymbol{w}\|=3$, what are the smallest and largest values of $\|\boldsymbol{v}-\boldsymbol{w}\|$?
   (b) What are the smallest and largest values of $\boldsymbol{v}\cdot\boldsymbol{w}$?

5. Find a combination $x_1\boldsymbol{w}_1+x_2\boldsymbol{w}_2+x_3\boldsymbol{w}_3$ that gives the zero vector. Are these vectors independent or dependent?

$$w_1=\begin{bmatrix}1\\2\\3\end{bmatrix} \qquad w_2=\begin{bmatrix}4\\5\\6\end{bmatrix} \qquad w_3=\begin{bmatrix}7\\8\\9\end{bmatrix}$$

6. Let the following matrices be defined.

$$A=\begin{bmatrix}4&7\\1&2\\5&6\end{bmatrix}, B=\begin{bmatrix}4&3&7\\1&2&7\\2&0&4\end{bmatrix}, C=\begin{bmatrix}3\\6\\1\end{bmatrix}, D=\begin{bmatrix}9&4&3&-6\\2&-1&7&5\end{bmatrix}$$

$$E=\begin{bmatrix}1&5&8\\7&2&3\\4&0&6\end{bmatrix}, F=\begin{bmatrix}3&0&1\\1&7&3\end{bmatrix}, G=\begin{bmatrix}7&6&4\end{bmatrix}$$

Answer the following questions regarding these matrices (each question is worth 2 points):

(a) What are the dimensions of the matrices?

(b) Identify the square, column, and row matrices.

(c) What is the determinant of matrix B?

(d) What is the trace of matrix E?

(e) What are the values of the elements: $a_{12}, b_{23}, d_{32}, e_{22}, f_{12}, g_{12}$.

(f) Perform the following operations or, if not well defined, explain why:

i. $E + B$

ii. $A + F$

iii. $B - E$

iv. $A \times B$

v. $B \times A$

vi. D^T

vii. $E \times B$

viii. C^T

ix. $A \times C$

x. $I \times B$

xi. $E^T \times E$

xii. $C^T \times C$

7. Let the following matrices be defined.

$$A = \begin{bmatrix} 5 & 6 & 6 & 8 \\ 2 & 2 & 2 & 8 \\ 6 & 6 & 2 & 8 \\ 2 & 3 & 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 17 & -9 & 12 & 16 \\ 17 & 8.75 & -11.75 & -16 \\ -4 & -2.25 & 2.75 & 4 \\ 1 & 0.75 & -0.75 & -1 \end{bmatrix}, C = \begin{bmatrix} -17 & -9 & 12 & 16 \\ 17 & 8.75 & -11.75 & -16 \\ -4 & -2.25 & 2.75 & 4 \\ 1 & 0.75 & -0.75 & -1 \end{bmatrix}$$

Which matrix, B or C, is an inverse to matrix A?

8. Let the following matrix and vectors be defined.

$$A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Answer the following questions (each question is worth 2 points):

(a) Write the matrix-vector system $Ax = b$ as a system of two linear equations of x_1 and x_2.

(b) Assuming that $b_1 = 1$ and $b_2 = 0$, solve this linear system for x_1 and x_2. Write this as a column vector.

(c) Assuming that $b_1 = 0$ and $b_2 = 1$, solve this linear system for x_1 and x_2. Write this as a column vector.

(d) Form a 2x2 matrix where the first column of the matrix is the answer vector from part (b) and the second column is the answer vector from part (c). Show that this matrix is the inverse of matrix A.