EAS 596, Fall 2019, Homework 4
Due Weds. 10/9, **3:30 PM**, Box outside Jarvis 326

Work all problems. Show all work, including any M-files you have written or adapted. Make sure your work is clear and readable - if the TA cannot read what you've written, they will not grade it. All electronic work (m-files, etc.) **must** be submitted through UBLearns (and submitted by the time class starts on the day it's due). Any handwritten work may be submitted in class. Each problem will be graded according to the following scheme: 2 points if the solution is complete and correct, 1 point if the solution is incorrect or incomplete but was using correct ideas, and 0 points if using incorrect ideas.

1. For which right-hand-sides (find a condition on b_1, b_2, b_3) are the following systems solvable?

   (a)
   $$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

2. Construct a 2-by-2 matrix whose nullspace equals its column space. This is possible.

3. Why does no 3-by-3 matrix have a nullspace that equals its column space? Hint: Think what has to be true if the nullspace equals the column space.

4. Find the reduced row echelon forms, rank, and nullity of the following matrices:

   (a) The 3-by-4 matrix with all entries equal 4
   (b) The 3-by-4 matrix with $a_{ij} = i + j - 1$

5. Fill out the following matrices so that they have rank 1.

(a)

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & & \\ 4 & & \end{bmatrix}$$

(b)

$$\begin{bmatrix} & 9 & \\ 1 & & \\ 2 & 6 & -3 \end{bmatrix}$$

(c)

$$\begin{bmatrix} a & b \\ c & \end{bmatrix}$$

6. For which numbers c and d do these matrices have rank 2?

(a)

$$\begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} c & d \\ d & c \end{bmatrix}$$

Note: You must use the same c and d values for both matrices. You must also verify that they are, in fact, rank 2.

7. If $\mathbf{V}$ is the subspace spanned by $(1, 1, 1)$ and $(2, 1, 0)$:

   (a) Find a matrix A that has $\mathbf{V}$ as its row space.
   (b) Find a matrix B that has $\mathbf{V}$ as its nullspace.
   (c) Multiply A and B.

8. A is an m-by-n matrix of rank r. Suppose there are right-hand-sides b for which $Ax = b$ has *no solution*.

   (a) What are all inequalities that must be true between m, n, and r?
   (b) How do you know that $A^T y = 0$ has solutions other than $y = 0$?

9. Show that the outer product between any two vectors, ab^T, always results in a rank 1 matrix. Hint: Show this holds for length-2 and length-3 vectors.