HW 3 P1

a)
$$\left[\begin{array}{cccc|c} 2 & 4 & 1 & 6 & 7 \\ -2 & 0 & 2 & 1 & 12 \\ -2 & 6 & 2 & 1 & 0 \\ -8 & -2 & 1 & 1 & -11 \end{array}\right]$$

$$\text{rref} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_0 = 3 \\ x_1 = -2 \\ x_2 = 9 \\ x_3 = 0 \end{array} \quad \text{or} \quad \begin{bmatrix} 3 \\ -2 \\ 9 \\ 0 \end{bmatrix}$$

b.)
$$\left[\begin{array}{cccc|c} 2 & 4 & 1 & 6 & 7 \\ -2 & 0 & 2 & 1 & 12 \\ -2 & 6 & 2 & 1 & 0 \\ 14 & -14 & 0 & -11 & -29 \end{array}\right]$$

$$\text{rref} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 11/6 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 7/3 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \text{Infinite \# of Solutions}$$

$$\Rightarrow \quad \underline{x} = \begin{bmatrix} 3 - 11/6\, x_3 \\ -2 \\ 9 - 7/3\, x_3 \\ x_3 \end{bmatrix} \quad \text{for any } x_3$$

Students must show at least one solution

c) $$\left[\begin{array}{cccc|c} 2 & 4 & 1 & 6 & 7 \\ -2 & 0 & 2 & 1 & 12 \\ -2 & 6 & 2 & 1 & 0 \\ 14 & -14 & 0 & -11 & 0 \end{array}\right]$$

$$\text{rref} = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 11/6 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 7/3 & 9 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Last row has $0=1 \Rightarrow$ $\underline{\text{No Solution}}$

HW 3 P3

a) G_{T_1}: $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ G_{T_2}: $\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$

G_{T_3}: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ G_{T_4}: $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

D_1: $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ D_2: $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

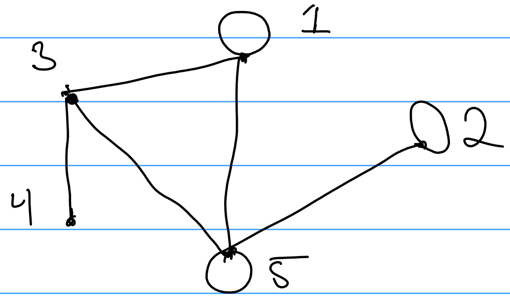
D_3: $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

$$D_4 : \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

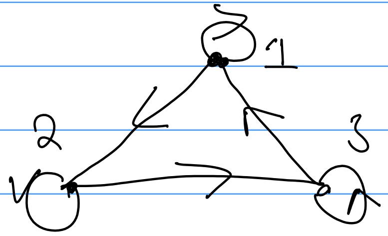
HW 3 P4

a, b, c, d, e, j, m → not adjacency matrices

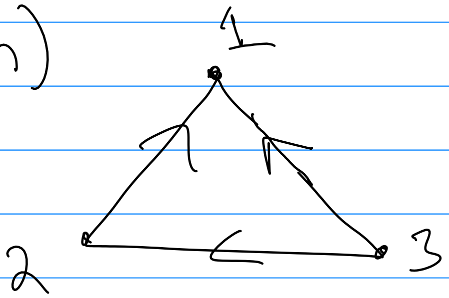
f)



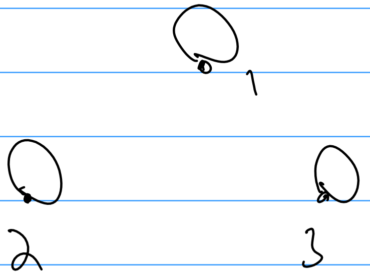
g)



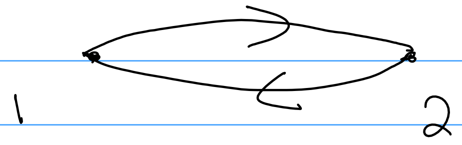
h)



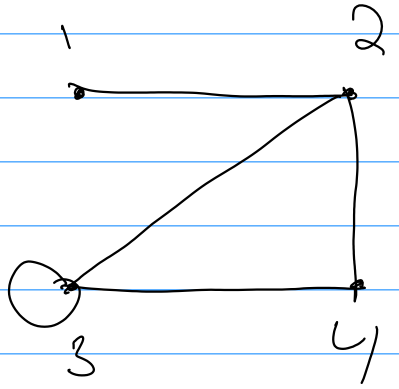
i)



k)



l)



HW 3 P5

a.) $P_1 = \begin{bmatrix} 0,2778 \\ 0,7222 \end{bmatrix}$ $P_2 = \begin{bmatrix} 0,3102 \\ 0,6892 \end{bmatrix}$

b.) $P_1 = \begin{bmatrix} 0,2222 \\ 0,4491 \\ 0,3194 \end{bmatrix}$ $P_2 = \begin{bmatrix} 0,2315 \\ 0,4491 \\ 0,3194 \end{bmatrix}$

c.) $P_1 = \begin{bmatrix} 0,3542 \\ 0,3333 \\ 0,3125 \end{bmatrix}$ $P_2 = \begin{bmatrix} 0,3559 \\ 0,3403 \\ 0,3038 \end{bmatrix}$

HW 3, P6

a) After 1 year: $\begin{bmatrix} 0.34 \\ 0{,}175 \\ 0.34 \\ 0.1450 \end{bmatrix}$ A & C tie

After 2 years! $\begin{bmatrix} 0.3555 \\ 0{,}1875 \\ 0{,}2875 \\ 0{,}1695 \end{bmatrix}$ Party A wins

b) After 100 years! $\begin{bmatrix} 0.36 \\ 0{,}20 \\ 0{,}24 \\ 0.20 \end{bmatrix}$

Party A: 36%   Party C: 24%

HW 3 P7

Let $k \in \mathbb{R}^1$

a) $[a, b, a]$ vectors in $\mathbb{R}^3$ w/ ① = ③

① ② ③

Addition: $[a,b,a] + [c,d,c] = [a+c, b+d, a+c]$

① ② ③

since ① = ③, closed under addition

Multiplication $k[a,b,a] = [ka, kb, ka]$

① ② ③

① = ③ closed under multiplication

$\Rightarrow$ A Subspac

b) Linear combinations of $[1,4,0]$ & $[2,2,2]$

$\Rightarrow a[1,4,0] + b[2,2,2]$

addition:

$$(a[1,4,0] + b[2,2,2]) + (c[1,4,0] + d[2,2,2])$$

$$= (a+c)[1,4,0] + (b+d)[2,2,2]$$

Closed under addition

Multiplication: $k(a[1,4,0] + b[2,2,2])$

$= (ka)[1,4,0] + (kb)[2,2,2] \Rightarrow$ Closed

$\Rightarrow$ A Subspac

c) $[a, b, c]$ w/ $a \leq b \leq c$

Addition: $[a, b, c] + [d, e, f]$

$= [a+d, b+e, c+f]$

If $a \leq b \leq c$ and $d \leq e \leq f$ then

$a+d \leq b+e \leq c+f \Rightarrow$ Closed

Multiplication $k[a, b, c] = [ka, kb, kc]$

Let $k = -1$ w/ $a = 1, b = 2, c = 3$

$\Rightarrow [ka, kb, kc] = [\underset{①}{-1}, \underset{②}{-2}, \underset{③}{-3}]$

As ① $\nleq$ ② $\nleq$ 3 $\Rightarrow$ $\underline{\underline{\text{Not}}}$ closed under multiplication

$\Rightarrow$ $\underline{\underline{\text{Not}}}$ a subspace

HW 3   P8

$$S = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

HW 3 P9

a) $\text{Span}(\{\underline{d}, \underline{e}\}) = a\begin{bmatrix} -8 \\ -7 \\ -5 \end{bmatrix} + b\begin{bmatrix} 16 \\ 9 \\ 15 \end{bmatrix}$

$= \begin{bmatrix} -8a + 16b \\ -7a + 9b \\ -5a + 15b \end{bmatrix} \Rightarrow$ Subset of $\mathbb{R}^3$

b) no $\Rightarrow$ need 5 vectors to span $\mathbb{R}^5$

c) $\underline{a} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$ $\underline{d} = \begin{bmatrix} -8 \\ -7 \\ -5 \end{bmatrix}$ $\underline{e} = \begin{bmatrix} 16 \\ 9 \\ 15 \end{bmatrix}$

To span $\mathbb{R}^3$ they must be linearly independent.

They are <u>not</u>: $2\underline{a} + (-1)\underline{d} = \underline{e}$

$\Rightarrow$ Do <u>not</u> span $\mathbb{R}^3$

d) Any <u>three</u> vectors of $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

will span a subspace of $\mathbb{R}^4$