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Chapter 6

Solving Linear Equations

Expression: is a "term" or set of terms that does contain a variable, but does not contain an equal sign

Equations: is a set of terms containing an equal sign between them

Expression $3x + 4$
Equation $3x + 4 = 12$

Solve the equation: we want to find the value of the variable that will make the equation a true statement.

$$x + 1 = 3$$

Conditional statement, because only $x = 2$ makes the equation true, and all other values make the equation false.

Check a solution, we substitute into the equation we are solving, to determine if it is true.

Equivalent expression: an expression that evaluates to the same numerical result if we plug in the same values of the variables.

Equivalent equation: an equation that has the same algebraic solution.

Addition Property of Equality (Subtraction Property of Equality)

$$a = b$$
$$a + c = b + c$$

$$a - c = b - c$$

$$x + 1 = 3$$
$$x + 1 - 1 = 3 - 1$$
$$x = 2$$

$$x - 7 = 15$$

Algebra = "the restoration"
Goal: to get the variable alone on one side

$$x - 7 + 7 = 15 + 7$$
$$x = 22$$

Multiplication Property of Equality (Division Property of Equality)

If I multiply one side of the equation by something, then I have to do the same thing to the other side

$$a = b$$
$$ac = bc$$

Division: multiplying by the reciprocal

Division by 3 is the same as multiplying by $\frac{1}{3}$

(division can't use zero)

$$a = b$$
$$\frac{a}{c} = \frac{b}{c}$$

As long as $c \neq 0$

$$3x = 12$$

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

Multiplication property handy when using fractions.

$$\frac{1}{2}x = \frac{7}{9}$$

Multiply by the denominator (one on the x side or the common denominator)
Multiply by 2

$$2 \cdot \frac{1}{2}x = 2 \cdot \frac{7}{9}$$

$$x = \frac{14}{9}$$

If you have a lot of terms in the equation, then the common denominator will help you eliminate the fractions from the equation.

$$\frac{1}{3}x + \frac{1}{4} = \frac{2}{7}x - \frac{11}{2}$$

Multiply by common denominator to eliminate the fractions. We'll come back to this.

After solving, check your solution in the **original problem**.

We may have to apply more than one property to get to a solution.

Goal: is still to get x alone on one side.

1) Combine like terms (addition/subtraction property)
2) Multiplication or division property to get just x

$$5x + 3 = 7x - 5$$

Get all x's on one side and all constants on the other.

Common to put x's on the left side.

$$5x + 3 - 7x = 7x - 7x - 5$$

$$-2x + 3 = -5$$
$$-2x + 3 - 3 = -5 - 3$$

$$-2x = -8$$

$$-\frac{2x}{-2} = -\frac{8}{-2}$$

$$x = 4$$

Check: 5(4)+3=23, 7(4)-5=23

$$6y + 7 = 18y - 1$$

$$6y + 7 - 6y = 18y - 6y - 1$$
$$7 = 12y - 1$$

$$7 + 1 = 12y - 1 + 1$$
$$8 = 12y$$

$$\frac{8}{12} = \frac{12y}{12}$$

$$\frac{2}{3} = y$$

Check: 6(2/3) =4, 4+7 =11, 18(2/3) =12, 12-1=11

Parentheses in equations

Simplify each side of the equation first (using our properties of expressions), and then use the properties of equality to solve.

$$2(x + 3) - 6 = 10$$

Steps:

1) Simplify both sides of the equation as much as possible
2) Use properties of equality to isolate x

Use distributive property to eliminate the parentheses

$$2x + 6 - 6 = 10$$
$$2x = 10$$
$$x = 5$$

Check: 2(5+3)-6 =2(8)-6=16-6=10

$$5 + 3(x + 7) = 26 - 6(5x + 11)$$

$$5 + 3x + 21 = 26 - 30x - 66$$

$$26 + 3x = -30x - 40$$

$$26 + 3x + 30x = -30x + 30x - 40$$
$$26 + 33x = -40$$

$$26 - 26 + 33x = -40 - 26$$

$$33x = -66$$

$$x = -\frac{66}{33} = -2$$

Check: 5+3(-2+7)=5+3(5)=5+15=20, 26-6(5*-2+11)=26-6(-10+11)=26-6(1)=26-6=20

Could increase the number of parentheses or introduce decimals.

Solving problems involving fractions.

$$\frac{2x}{3} = \frac{32}{6}$$

Solve like a proportions
Cross-multiply and then solve

$$12x = 96$$
$$x = \frac{96}{12} = 8$$

$$\frac{2}{3} + \frac{x}{4} = \frac{28}{6}$$

Steps:

1) Find a common denominator to eliminate the fractions
2) Simplify both sides
3) Use properties of equality to solve for (isolate) x

$$\frac{12}{1}\left(\frac{2}{3}+\frac{x}{4}\right)=\frac{12}{1}\left(\frac{28}{6}\right)$$

$$\frac{12}{1}\cdot\frac{2}{3}+\frac{12}{1}\cdot\frac{x}{4}=\frac{12}{1}\cdot\frac{28}{6}$$

$$8+3x=56$$

All the fractions are gone

$$-8+8+3x=56-8$$
$$3x=48$$
$$x=16$$

Check: 2/3 +16/4 =2/3+4 =2/3+12/3=14/3, 28/6=14/3

Convert any mixed numerals to improper fractions

$$5x+\frac{(6x-8)}{14}+\frac{(10x+6)}{6}=43$$

Find a common denominator: 2x7, 2x3: 2x7x3=42

$$42(5x)+\frac{42}{1}\cdot\frac{6x-8}{14}+\frac{42}{1}\cdot\frac{10x+6}{6}=42(43)$$

$$42(5x)+3(6x-8)+7(10x+6)=42(43)$$

$$210x+18x-24+70x+42=1806$$

$$298x+18=1806$$
$$298x+18-18=1806-18$$
$$298x=1788$$

$$x=\frac{1788}{298}=6$$

Do check: the more complex the problem, the important checking is.

Building up to word problems/applications

+	−	×	÷	=
plus	minus	times	divide	equal or equals
increased by	decreased by	product	quotient	is or are
added to	subtract	multiply by	divided by	is equal to
more than	less than	double or twice		result is
sum of	difference	triple or thrice		
	subtract from			

Divided into any sort of verb

The sum of a number and the number decreased by six is five.

X + (x-6) =5

"number" = variable

$$x + (x - 6) = 5$$

Twice the sum of a number and five is eighty.

=80

2(x+5)

$$2(x + 5) = 80$$

The product of a number decreased by seven and the same number increased by five is thirteen.

$$(x - 7)(x + 5) = 13$$

Seven less than the quotient of a number and six is two.

$$\frac{x}{6} - 7 = 2$$

Note the order here.

Seven less than a number is $x - 7$
A number subtracted from 7 is $7 - x$
A number minus 7 is $x - 7$

A number divided by 7 is $\frac{x}{7}$

A number divided into 7 is $\frac{7}{x}$

Application problems:

1) Read the problem more than once (take one sentence at a time, even a clause at time)
2) Draw a diagram or a picture
3) Select a variable or variables and write down what they mean (generally, the last sentence in a word problem tells you what the variable represents)
4) Write an equation relating the information in the problem. May come from the problem itself, or it may involve a formula.
5) Solve the equation
6) Check the solution. Do a logic check. Add units.

Distribute $4950 among John, Maria, and Betsy so that Maria receives twice as much as John and Betsy receives three times as much as John.

$$x = amount\ of\ money\ going\ to\ John$$

$$2x = amount\ of\ money\ going\ to\ Maria$$

$$3x = amount\ of\ money\ going\ to\ Betsy$$

$$x + 2x + 3x = 4950$$

$$6x = 4950$$
$$x = 825$$

John gets $825, Maria get $1650, and Betsy gets $2475.

The total cost of 20 boards is $166. One size costs $6.50, and the second size costs $9.50. How many boards are purchased at each price?

$$x = number\ of\ boards\ of\ first\ size, \quad \$6.50\ each$$
$$20 - x = y = number\ of\ boards\ of\ second\ size\ , \$9.50\ each$$

$x + y = 20$

$$6.50x + 9.50(20 - x) = 166$$

$$6.5x + 190 - 9.5x = 166$$
$$-3x + 190 = 166$$

$$-3x + 190 - 190 = 166 - 190$$
$$-3x = -24$$

$$x = -\frac{24}{-3} = 12$$

12 boards of the first size, and 8 boards of the second size.

6.7 Solving formulas:
Formula for the area of a triangle: $A = \frac{1}{2}bh$

Have area, and have base, but I don't have the height?

$$A = \frac{1}{2}bh$$

$$2 \times A = 2 \times \frac{1}{2}bh$$

$$2A = bh$$

$$\frac{2A}{b} = h$$

Using properties of equality to solve for the indicated variable. Purpose is to get comfortable working properties of equality (and algebraic expressions) with symbols or variables.

6.8 working with formulas in problems
6.9 same but formulas have fractions in them: $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{total}}$

$$R_1 = 2$$
$$R_2 = 3$$

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6} = \frac{1}{R_{total}}$$

$$\frac{6}{5} = \frac{R_{total}}{1}$$
$$R_{total} = 1.2$$