10/30/2020

Chapter 12: Geometry
2D today

An angle is defined by two sides: 1) initial side, 2) terminal side

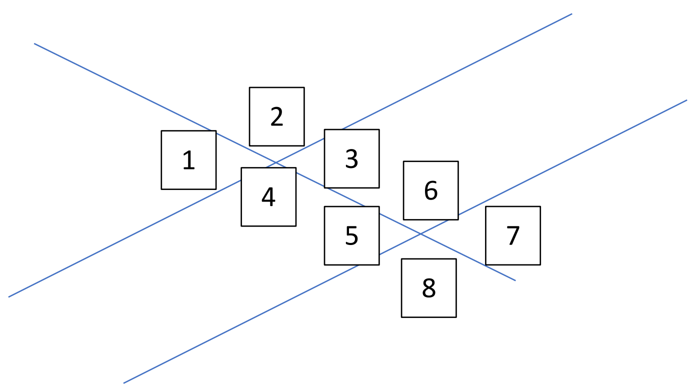
Standard position: an angle is drawn with the initial side on the positive x-axis.
For now, we will use degrees to measure the angle. 360-degrees is a circle.
Straight angle is 180-degree angle (half the circle)
90-degree angle is ¼ of a circle, and this is called a right angle.
Anything between 90 and 180 is called obtuse
Anything between 0 and 90 is called acute

Point where the two sides meet, is called the vertex.

∠

$\angle 1$

Create an angle, we need two lines: terminology of angles from the intersection of two lines.



Vertical angles are opposite each other across the vertex:
2 and 4 are vertical angles
1 and 3 are vertical angles
Vertical angles are always equal.

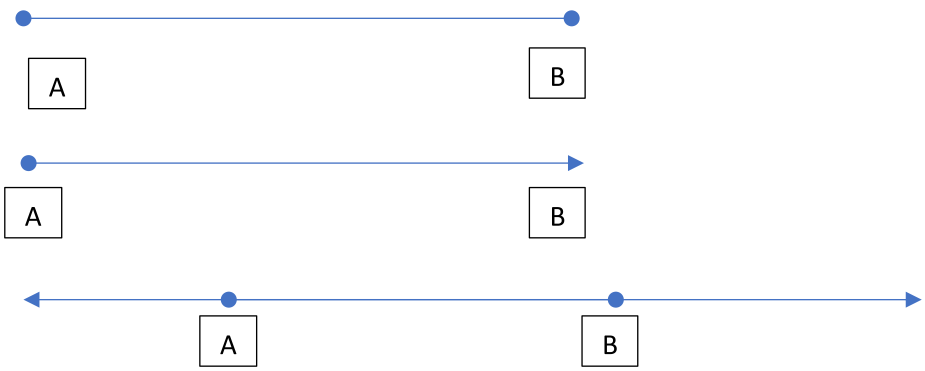
Adjacent angles share a side.

Parallel lines intersected with a transversal (a line that crosses both parallel lines) creates 8 angles.

Interior angles (3 and 6, or 4 and 5): add to 180-degrees (supplementary)
Alternate interior angles (3 and 5, or 4 and 6): these angles are equal
Alternate exterior angles (2 and 8, or 1 and 7): these angles are equal
Exterior angles (2 and 7, or 1 and 8): these add to 180 (supplementary)
Corresponding angles (2 and 6, or 1 and 5, or 4 and 8, or 5 and 7) these angles are equal

Supplementary angles add to 180 (straight angle)
Complementary angles add to 90 (right angle)

A line (segment) is defined by two points



Top one: line segment $\overline{AB}$
Middle one: ray $\overrightarrow{AB}$
Bottom one: line $\overleftrightarrow{AB}$

Polygons shapes with 3 or more (straight) sides that form a closed region.
If you name the vertices you can refer to the shape.

$$\triangle ABC$$

3-sides : triangle
4-sides : quadrilateral
5-sides : pentagon
6-sides : hexagon
7-sides : heptagon
8-sides : octagon
9-sides : nonagon
10-sides: decagon

Regular vs irregular
All the sides and angles are the same size in a regular polygon

Polygon: the sum of the interior angles of a n-gon, 180(n-2)

Quadrilaterals: shapes with 4 sides

Trapezoid: two sides are parallel

$$A_T = \frac{1}{2}h(b_1 + b_2)$$

Parallelogram: two sets of parallel sides: requires two sets of equal sides (opposite sides are equal), two sets of equal angles (opposite angles are equal).

$$P = 2b + 2a$$
$$A = bh$$

Rectangle: two sets of parallel sides that intersect at a right angle: opposite sides are equal, and all angles are equal.

$$A = lw$$
$$P = 2l + 2w$$

Square: two sets of parallel sides, that intersect at a right angle, all sides are equal: all angles are equal (regular quadrilateral)

$$A = s^2$$
$$P = 4s$$

Rhombus: two sets of parallel sides, but all sides are equal: all opposite angles are equal

$$A = bh$$
$$P = 4b$$

(Kite: two sets of equal sides (that are not parallel))

Triangles:

A triangle that has no sides or angles alike: scalene triangle

Isosceles triangles have two sides (and two angles) that are the same size

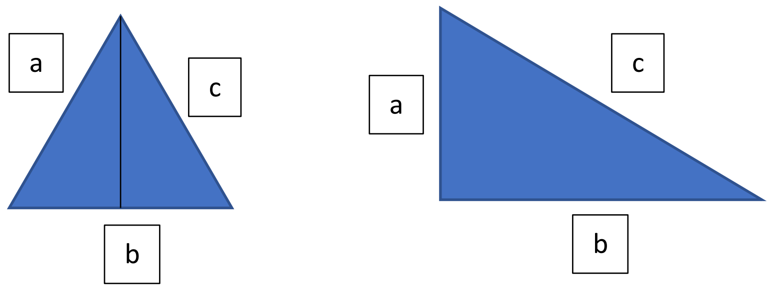
Equilateral triangle: all sides and all angles are equal (regular)

Acute triangle: all the angles in the triangle are less than 90-degrees

Right triangle: one angle which is 90-degrees (the remaining two angles are complementary)

Obtuse triangle: one angle is more than 90 degrees

The hypotenuse is the longest side of a right triangle, opposite the 90-degree angle. The other two are called legs, and these are both of the shorter sides.



$$A_{tri} = \frac{1}{2}bh$$

Heron's formula

$$s = \frac{1}{2}(a + b + c)$$
$$A_{tri} = \sqrt{s(s-a)(s-b)(s-c)}$$

Right triangle with sides 3, 4 and 5.

$$A = \frac{1}{2}(3)(4) = 6$$

$$s = \frac{1}{2}(3 + 4 + 5) = 6$$
$$A = \sqrt{6(6-3)(6-4)(6-5)} = \sqrt{6(3)(2)(1)} = \sqrt{36} = 6$$

Pythagorean theorem: only for right triangle

$$a^2 + b^2 = c^2$$

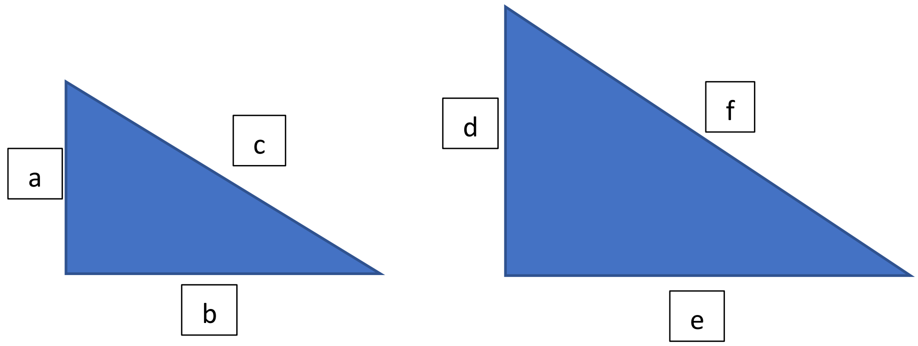
$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

Pythagorean triples:

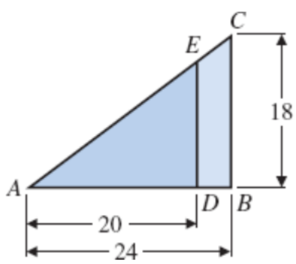
3-4-5
5-12-13
7-24-25
8-15-17
Also, any multiples of these will work

Similar polygons

Two polygons are considered similar if all corresponding angles are equal.
Is that the corresponding sides are ratios of each other.



$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$



$$\frac{smaller}{larger} = \frac{20}{24} = \frac{DE}{18}$$

Solve using proportions (ch7)
Use Pythagorean theorem to solve for missing hypotenuse

Circles

Circumference (like the perimeter) = $2\pi r = \pi d$
r is the radius, d is the diameter, and $\pi \approx 3.1415926 \ldots \approx \frac{22}{7}$

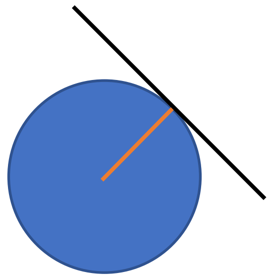
radius is the distance from the center of a circle to any point on the circle.
Diameter is the distance between two points on a circle that passes through the center = 2r

$$A = \pi r^2$$

Central angle is an angle whose vertex is the center of the circle and the sides of the angle are radii

Chord (secant) is any line that connects two points on a circle (typically does not pass through the center)

Tangent is a line that touches the circle at a single point



A radius that intersects with a tangent line intersects at a right angle.

If an angle is created by two chords inside a circle, this is called an inscribed angle.

The portion of the circle bounded by the inscribed angle is called an inscribed arc.

Radians

Is a way to measure an angle relative to the circumference

The circumference of a circle with radius = 1 is 2π.
The degree measure of the circle is 360-degrees.

2π radians is all the way around the circle = 360-degrees
π radians is halfway around the circle = 180-degrees
$\frac{\pi}{2}$ radians is ¼ of the way around a circle = 90-degrees

$\frac{\pi}{3}$ = 60 degrees
$\frac{\pi}{4}$ = 45-degrees

$\frac{\pi}{6}$ = 30-degrees

To convert from degrees to radians $\frac{\pi}{180}$

To convert from radians to degrees $\frac{180}{\pi}$

The formula for the length of an arc of circle, requires using radians

Length of an arc of a circle of radius r:

$$s = r\theta$$

θ is the angle in radians

Arc that subtends an angle of 45-degrees of a circle, if the radius is 6 inches, how long is the arc?

$$s = 6\left(\frac{\pi}{4}\right) = 4.71\ inches$$

Sector measures the area of the pizza slice:

$$A = \frac{1}{2}r^2\theta$$

θ is the angle in radians, r is the radius

Regular polygons, the area is usually calculated by splitting the shape up into triangles.