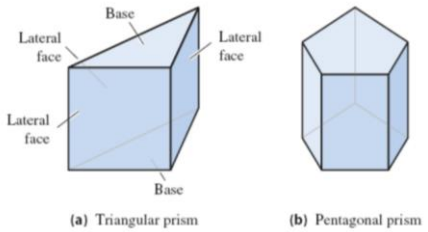
11/06/2020

Chapter 12
3D Geometry

Prisms

Prism is a shape with a base that is a polygon and the opposite face is the same shape, and two bases are connected with lines.

Volume of prism: $B * h$
B = area of the base
h = perpendicular height (measured from the base)

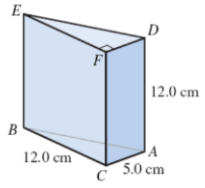


Surface area
Lateral surface area is the area of the sides of the shape (does not include the bases)
Total surface area includes the lateral surface and the area of the base(s)

Rectangular prism (special case) = rectangular box

$$V = lwh$$

Surface Area = $2lw + 2lh + 2wh$



Area of the base: $\frac{1}{2}bh = \frac{1}{2}(12)(5) = 30\ cm^2$
Vol = $Bh = 30(12) = 360\ cm^3$

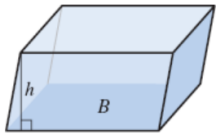
Lateral surface area = $12(12) + 5(12) + 13(12) = 360\ cm^2$

$$12^2 + 5^2 = 144 + 25 = 169 = 13^2$$

Total surface area = Lateral surface + areas of the bases = $360 + 30 + 30 = 420\ cm^2$

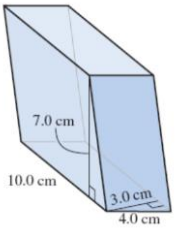
Right prisms have sides that are perpendicular to the bases

But to do “slanted” prisms the height is still calculated as if it were perpendicular to the base, so it may not be the length of a side.



height is the distance between two parallel faces

Parallelepiped
(3D version of a parallelogram ~ slanty box)



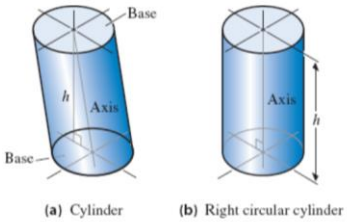
Volume = 10(7)(3)=210 cm^3

Area of parallelogram is bh =10(3)

Cylinder
Prism has a polygon for a base
(circular) Cylinder has a circle for a base

Almost everything else is identical except for how we calculate the area of the base

Fundamentally still volume is still Bh



Right cylinders have sides that are perpendicular to the bases. If they are not right cylinders, the height is the perpendicular distance between the bases.

Soda can: 6 in. high, 3 in diameter

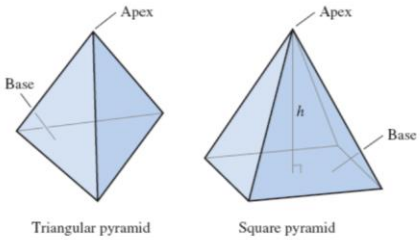
Area of the base is a circle. Area = $\pi r^2 = \pi(1.5)^2 = 7.1\ in^2$
Volume = $Bh = \pi(1.5)^2(6) = 42.4\ in^3$

Lateral surface area = circumference of the circle * height of the cylinder = $2\pi rh$

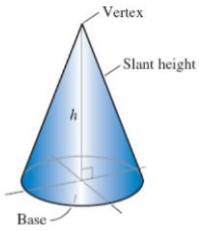
Total surface area is $2\pi rh + 2\pi r^2$

Lateral surface area of the pop can: $2\pi(1.5)6 = 56.5\ cm^2$

Total surface area = 56.5+2(7.1)=70.7 cm^2



Pyramid has a single base which is a polygon and the sides rise to meet at a point. (a right pyramid has an apex directly above the geometric center of the base)
Cone has a single base which is a circle (round), rises to a single point.



right circular cone = implies that the vertex is directly above the center of the circle.

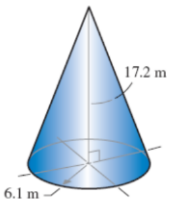
Volume for a pyramid or a cone = $\frac{1}{3}Bh$

Circular cone volume = $\frac{1}{3}\pi r^2 h$

Square pyramid = $\frac{1}{3}s^2 h$
Triangular pyramid = $\frac{1}{3}\left(\frac{1}{2}ba\right)h = \frac{1}{6}abh$ (a = altitude of the triangular base)

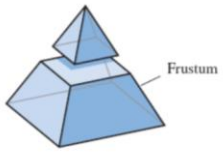
Surface area of pyramid = all sides are triangles, and so calculate the area of each triangle and add up. The number of faces depends on the shape of the base. The "height" of the triangles are called the "slant height".

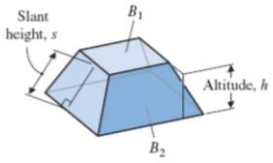
Lateral surface area for a cone = πrs
s = slant height



Volume $= \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi(6.1)^2 17.2 = 670.2\ m^3$
Lateral surface area = $\pi rs = \pi(6.1)\sqrt{6.1^2 + 17.2^2} = 349.7\ m^2$
Total surface area = $349.7 + (\pi r^2) = 349.7 + \pi(6.1)^2 = 466.6\ m^2$

Frustrum
A cone or pyramid which has the top lobbed off.
There are two bases of the same shape, but they are similar (different sizes, but same configuration)





$V = \frac{1}{3}h\,(B_1 + B_2 + \sqrt{B_1 B_2})$

where h is the altitude and B_1 and B_2 are the areas of the bases.

The *lateral surface area of the frustum of a pyramid* is

$A = \frac{1}{2}s(P_1 + P_2)$

$P_1\ and\ P_2$ are perimeters of the top and bottom bases

Sphere

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area = $4\pi r^2$

Ball of radius 11 in. What is the volume and surface area of the ball?

Vol= $\frac{4}{3}\pi(11)^3 = 5575.3\ in^3$
SA=$4\pi(11)^2 = 1520.5\ in^2$

Hemisphere = ½ sphere

Degrees with decimal places
Degrees -minutes-seconds (DMS)
60 minutes in a degree (arcminutes), 60 seconds in a minute (arcseconds)

$$36°17'43''$$

$$36 + \frac{17}{60} + \frac{43}{3600} = 36.295277777\ldots$$

1/3600 = 0.00027777…

$$43.760000°$$

$$43° +$$

$$0.76(60) = 45.6'$$

$$43°45' +$$

$$0.6(60) = 36$$

$$43°45'36''$$

The end