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Chapter 7: Ratios and Proportions

Ratio is a relationship between two values

(rate)

$$\frac{A}{B}$$
$$A : B$$
$$A \text{ to } B$$

Simplifying a ratio: put it in lowest terms

$$45 : 15 = 3 : 1$$
$$\frac{45}{15} = \frac{3}{1}$$

$$5\frac{1}{3} \text{ to } 2\frac{1}{7}$$

$$\frac{16}{3} \text{ to } \frac{15}{7}$$

$$\frac{16}{3} \div \frac{15}{7} = \frac{16}{3} \times \frac{7}{15} = \frac{112}{45} = 2.4\bar{8} : 1$$

Ratios become rates when the values being compared are in different units.

Proportions

Relationship between ratios that allows us to solve for some missing quantity.

$$\frac{A}{B} = \frac{C}{D}$$

$$A : B :: C : D$$

$$A \text{ to } B \text{ as } C \text{ to } D$$

Ratios in a proportion are always equal.

$$A : B = C : D$$

In a proportion, the means are the two numbers in the middle: B and C
The extremes are the two numbers on the ends: A and D

$$\frac{x}{3} = \frac{8}{12}$$

Proportion problem with an unknown

What does x have to be in order for this expression to be a proportion?

Set the product of the means = to the product of the extremes: cross-multiply

$$12x = 24$$

$$x = 2$$

$$\frac{2}{3} = \frac{8}{12}$$

If 125 bolts cost \$16.50, then how much do 75 bolts cost?

$$\frac{125}{16.50} = \frac{75}{x}$$

Cross multiply

$$1237.5 = 125x$$

Divide by # in front of x

$$x = \$9.90$$

$$\frac{125}{75} = \frac{16.5}{x}$$

Percent problems:

Rate, base, and part

$$\frac{R}{100} = \frac{P}{B}$$

Unit conversions

$$30 \frac{mi}{h} \text{ to } x \frac{feet}{sec}$$

$$\frac{30 \text{ mi} \times (5280 \frac{ft}{mi})}{h \times 60 \frac{min}{h} \times 60 \frac{sec}{min}} = \frac{158,400 \text{ ft}}{3600 \text{ sec}} = 44 \frac{ft}{sec}$$

Direct and Indirect Variation

Direct variation: $\frac{y_1}{x_1} = \frac{y_2}{x_2}$

$$y = kx$$

k is the constant rate of change

Map scale:

For every inch on the map, the miles are equivalent to some value m .

1 inch = 32 miles

Measure on the map: $2\frac{3}{8}$ in.

$$\frac{\text{inches on map}}{\text{distance in world}} = \frac{1}{32} = \frac{2\frac{3}{8}}{x}$$

$$\frac{1}{32} = \frac{2\frac{3}{8}}{x}$$

$$x = \left(2\frac{3}{8}\right) \times 32 = \frac{19}{8} \times \frac{32}{1} = 76$$

Inverse Variation

$$x_1 y_1 = x_2 y_2$$

$$\frac{1}{2} \times 2 = \frac{1}{3} \times 3$$

$$2 \times 8 = 1 \times 16$$

Seesaw

Balanced on a base, long vertical board balanced on base

$$F_1 \times d_1 = F_2 \times d_2$$

$$30 \text{ kg} \times 2 \text{ m} = 20 \text{ kg} \times d_2$$

$$60 \text{ kg m} = 20 d_2 \text{ kg m}$$

$$3 = d_2$$

Alternatively: $y = \frac{k}{x}$