Math 266, Quiz #10, Fall 2021 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Let $W = span\left\{\begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}\right\}$. Find an orthogonal basis for $W^{\perp}$.

u $\qquad$ $v, w \in W^{\perp}$

$u \cdot v = 0$

$u \cdot w = 0$

$v \neq w$

$v \cdot w = 0$

$v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\qquad$ $a - 3b + 5c = 0$, $c = 0$, $a = 3b$, $b = 1$ $\qquad$ $v = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

$w = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$ $\qquad$ $d - 3e + 5f = 0$

$3d + e + 0 = 0$

$\begin{bmatrix} 1 & -3 & 5 \\ 3 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & -3/2 \end{bmatrix}$

$d \quad x_1 = -1/2\, x_3$

$e \quad x_2 = 3/2\, x_3$

$f \quad x_3 = x_3$

$w = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

$W^{\perp} = \left\{\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}\right\}$

2. An orthogonal basis for R^2 is $\left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$. Find an expression for $\vec{x} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ in this basis. Do not use inverse matrices (use projections).

$x_B =$

$b_1 \cdot \vec{x} = 3 + 14 = 17$

$b_2 \cdot \vec{x} = -6 + 7 = 1$

$c_1 = \dfrac{b_1 \cdot \vec{x}}{b_1 \cdot b_1} = \dfrac{17}{5}$

$c_2 = \dfrac{b_2 \cdot \vec{x}}{b_2 \cdot b_2} = \dfrac{1}{5}$

$x_B = \begin{bmatrix} 17/5 \\ 1/5 \end{bmatrix}$

3. Let $W = span\left\{\begin{bmatrix}1\\-2\\-1\\2\end{bmatrix}, \begin{bmatrix}-4\\1\\0\\3\end{bmatrix}\right\}$. Given $\vec{y} = \begin{bmatrix}3\\-1\\1\\13\end{bmatrix}$, decompose this vector into $\vec{y}_{\parallel}$ in W and $\vec{y}_{\perp}$ in $W^{\perp}$.

$$\underset{u}{} \quad \underset{v}{}$$

$$(1)(-4) + (-2)(1) + 0 + (2)(3) = -4 - 2 + 6 = 0$$

$$\frac{u \cdot y}{u \cdot u} u = \frac{3+2-1+26}{1+4+1+4}\begin{bmatrix}1\\-2\\-1\\2\end{bmatrix} = \frac{30}{10}\begin{bmatrix}1\\-2\\-1\\2\end{bmatrix} = 3\begin{bmatrix}1\\-2\\-1\\2\end{bmatrix} = \begin{bmatrix}3\\-6\\-3\\6\end{bmatrix}$$

$$\frac{v \cdot y}{v \cdot v} v = \frac{-12-1+0+39}{16+1+0+9}\begin{bmatrix}-4\\1\\0\\3\end{bmatrix} = \frac{26}{26}\begin{bmatrix}-4\\1\\0\\3\end{bmatrix} = \begin{bmatrix}-4\\1\\0\\3\end{bmatrix}$$

$$\vec{y}_{\parallel} = \begin{bmatrix}3\\-6\\-3\\6\end{bmatrix} + \begin{bmatrix}-4\\1\\0\\3\end{bmatrix} = \begin{bmatrix}-1\\-5\\-3\\9\end{bmatrix}$$

$$\vec{y}_{\perp} = \underbrace{\begin{bmatrix}3\\-1\\1\\13\end{bmatrix}}_{\vec{y}} - \underbrace{\begin{bmatrix}-1\\-5\\-3\\9\end{bmatrix}}_{\vec{y}_{\parallel}} = \begin{bmatrix}3+1\\-1+5\\1+3\\13-9\end{bmatrix} = \begin{bmatrix}4\\4\\4\\4\end{bmatrix} \in W^{\perp}$$

4. A basis for R^3 is $\left\{\begin{bmatrix}1\\2\\5\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\2\end{bmatrix}\right\}$. Use Gram-Schmidt to find an orthogonal basis for the space.

$$b_1 = \begin{bmatrix}1\\2\\5\end{bmatrix}$$

$$b_2 = \begin{bmatrix}1\\0\\1\end{bmatrix} - \frac{1+0+5}{1+4+25}\begin{bmatrix}1\\2\\5\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix} - \frac{6}{30}\begin{bmatrix}1\\2\\5\end{bmatrix} =$$

$$\begin{bmatrix}1\\0\\1\end{bmatrix} - \begin{bmatrix}1/5\\2/5\\1\end{bmatrix} = \begin{bmatrix}4/5\\-2/5\\0\end{bmatrix} \to \begin{bmatrix}4\\-2\\0\end{bmatrix} \to \begin{bmatrix}2\\-1\\0\end{bmatrix}$$

$$b_3 = \begin{bmatrix}1\\-1\\2\end{bmatrix} - \frac{1-2+10}{1+4+25}\begin{bmatrix}1\\2\\5\end{bmatrix} - \frac{2+1+0}{4+1+0}\begin{bmatrix}2\\-1\\0\end{bmatrix} = \begin{bmatrix}1\\-1\\2\end{bmatrix} - \frac{9}{30}\begin{bmatrix}1\\2\\5\end{bmatrix} - \frac{3}{5}\begin{bmatrix}2\\-1\\0\end{bmatrix} =$$

$$\begin{bmatrix}-1/2\\-1\\1/2\end{bmatrix} \to \begin{bmatrix}-1\\-2\\1\end{bmatrix}$$

$$\text{basis} = \left\{\begin{bmatrix}1\\2\\5\end{bmatrix}, \begin{bmatrix}2\\-1\\0\end{bmatrix}, \begin{bmatrix}-1\\-2\\1\end{bmatrix}\right\}$$

5. Use least squares to find a linear regression equation $\beta_0 + \beta_1 x = y$ for the data shown in the table below. Be sure to write the final regression equation.

x	8	5	2	1	6	3
y	41	33	29	28	38	30

$$A = \begin{array}{cc} \beta_1 & \beta_0 \end{array}$$
$$A = \begin{bmatrix}8 & 1\\5 & 1\\2 & 1\\1 & 1\\6 & 1\\3 & 1\end{bmatrix} \quad \vec{b} = \begin{bmatrix}41\\33\\29\\28\\38\\30\end{bmatrix}$$

$$(A^TA)^{-1}A^Tb = X = \begin{bmatrix}37/19\\476/19\end{bmatrix} \approx \begin{bmatrix}1.94\\25.05\end{bmatrix}$$

$$Y = 1.94X + 25.05$$