Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Write the system of equations $\begin{cases} x_1 \quad\quad - 3x_3 = 8 \\ 2x_1 + 2x_2 + 9x_3 = 7 \\ \quad\quad x_2 + 5x_3 = -2 \end{cases}$ as an augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right]$$

2. Perform row operations on the augmented matrix you found in #1, and put the matrix in echelon form.

$-2R_1 + R_2 \rightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right]$$

$-2R_2 + R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right]$$

$\frac{1}{5}R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad \text{echelon form}$$

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$-5R_3 + R_2 \rightarrow R_2$
$3R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

3. Is the system consistent or inconsistent? If it's consistent, is it dependent or independent? If the solution is consistent and independent, write the solution as a vector.

consistent, independent

$$\vec{x} = \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$

4. Write the system in #1 as a vector equation.

$$x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix}$$

5. Write the system in #1 as a matrix equation.

$$\begin{bmatrix} 1 & 0 & -3 \\ 2 & 2 & 9 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ -2 \end{bmatrix}$$

$$A \qquad \vec{x} \quad = \quad \vec{b}$$