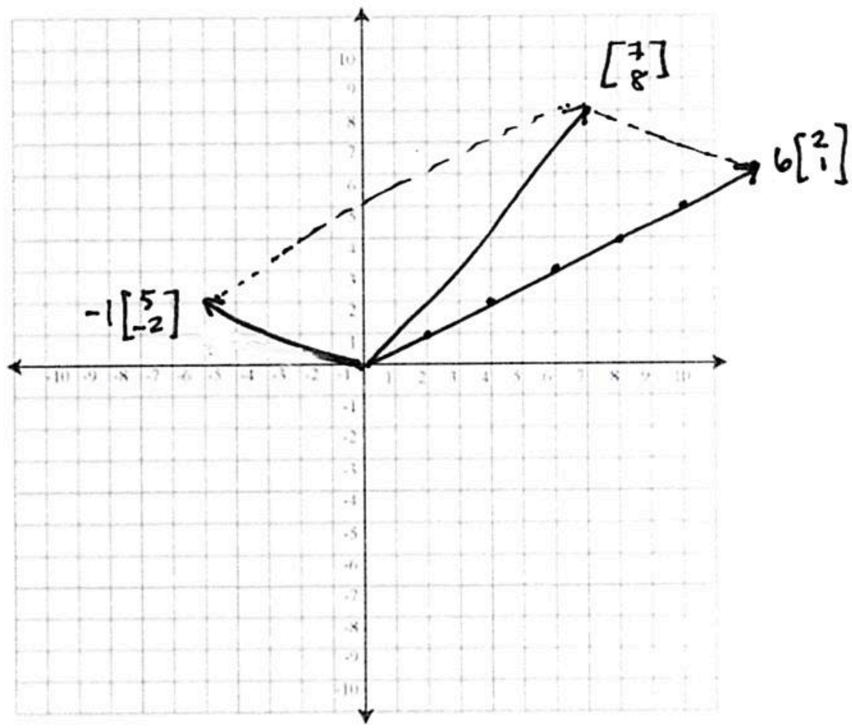
Math 266, Quiz #2, Fall 2021   Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. The solution to the system $x_1\begin{bmatrix}2\\1\end{bmatrix}+x_2\begin{bmatrix}5\\-2\end{bmatrix}=\begin{bmatrix}7\\8\end{bmatrix}$ is $\vec{x}=\begin{bmatrix}6\\-1\end{bmatrix}$. Represent the solution graphically on the graph below.



2. Solve the system of equations $\begin{cases}x_1+2x_2-3x_3=0\\2x_1+x_2-3x_3=0\\-x_1+x_2\qquad=0\end{cases}$ and write the solution in parametric form.

$$\left[\begin{array}{ccc|c}1&2&-3&0\\2&1&-3&0\\-1&1&0&0\end{array}\right]$$

$-2R_1+R_2\to R_2$
$R_1+R_3\to R_3$

$$\left[\begin{array}{ccc|c}1&2&-3&0\\0&-3&3&0\\0&3&-3&0\end{array}\right]$$

$R_2+R_3\to R_3$

$$\left[\begin{array}{ccc|c}1&2&-3&0\\0&-3&3&0\\0&0&0&0\end{array}\right]$$ dependent

$-\frac{1}{3}R_2\to R_2$

$$\left[\begin{array}{ccc|c}1&2&-3&0\\0&1&-1&0\\0&0&0&0\end{array}\right]$$

$-2R_2+R_1\to R_1$

$$\left[\begin{array}{ccc|c}1&0&-1&0\\0&1&-1&0\\0&0&0&0\end{array}\right]$$

$x_1-x_3=0$
$x_2-x_3=0$
$x_3=x_3$

$\downarrow$

$x_1=x_3$
$x_2=x_3$
$x_3=x_3$

$$\vec{x}=\begin{bmatrix}1\\1\\1\end{bmatrix}x_3$$

3. Determine if the following sets of vectors are linearly independent. Explain your reasoning.

a. $\left\{\begin{bmatrix}1\\6\\2\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}\right\}$ yes. 2 vectors not multiple of each other

b. $\left\{\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}, \begin{bmatrix}-1\\1\end{bmatrix}\right\}$ no. more vectors than available coordinates (dimensions)

c. $\left\{\begin{bmatrix}1\\2\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\3\\1\end{bmatrix}, \begin{bmatrix}5\\0\\1\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\3\\3\end{bmatrix}\right\}$ no

if you solve the homogeneous system

$$x_1\begin{bmatrix}1\\2\\1\\0\end{bmatrix} + x_2\begin{bmatrix}-1\\1\\3\\1\end{bmatrix} + x_3\begin{bmatrix}5\\0\\1\\2\end{bmatrix} + x_4\begin{bmatrix}3\\-1\\3\\3\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

the system is dependent (infinite solutions)

so the vectors are also dependent.