Math 266, Quiz #6, Fall 2021 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Determine if the following sets are subspaces. Be sure to check all the necessary conditions or find a counterexample.

a. $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} | xy \geq 0 \right\}$. not a subspace

fails addition $\vec{u} = \begin{bmatrix} -1 \\ -6 \end{bmatrix}$ & $\vec{v} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ but

$(-1)(-6) = 6 \geq 0$ $(7)(2) = 14 \geq 0$

$\vec{u} + \vec{v} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$ $(6)(-4) = -24 \not\geq 0$

not in the set. not closed under addition

b. $H = \left\{ \begin{bmatrix} 3s+4t \\ s \\ 2s-3t \\ 5t \end{bmatrix}, s,t \in R \right\}$ 1) $\vec{0}$ in set if $s=t=0$

2) $\begin{bmatrix} 3s+4t \\ s \\ 2s-3t \\ 5t \end{bmatrix} + \begin{bmatrix} 3u+4v \\ u \\ 2u-3v \\ 5v \end{bmatrix} = \begin{bmatrix} 3(s+u)+4(t+v) \\ s+u \\ 2(s+u)-3(t+v) \\ 5(t+v) \end{bmatrix}$ ✓ $s,t,u,v \in \mathbb{R}$

this is a subspace

3) $k \begin{bmatrix} 3s+4t \\ s \\ 2s-3t \\ 5t \end{bmatrix} = \begin{bmatrix} 3(ks)+4(kt) \\ ks \\ 2(ks)-3(kt) \\ 5kt \end{bmatrix}$ ✓ $ks, kt \in \mathbb{R}$

c. The set of all complex numbers.

This is a subspace $a+bi$

1) if $a=b=0$, 0 in set

2) $a+bi + c+di = (a+c) + (b+d)i$ $a,b,c,d \in \mathbb{R}$

3) $k(a+bi) = ka + kbi$ $k,a,b \in \mathbb{R}$

d. Polynomials of the form $p(t) = 1 + bt + ct^2$.

not a subspace

0 is not in the space. if $b=c=0$ $p(t)=1$, not 0.

also fails addition and scalar multiplication

$p(t) = 1 + bt + ct^2$, $q(t) = 1 + dt + et^2$

$p+q = 2 + (b+d)t + (c+e)t^2$

Constant is not 1, so not in set