

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Suppose matrix A is a 9×7 matrix with 5 pivot columns. Determine the following.

$$\dim \text{Col } A = \underline{5}$$

$$\dim \text{Nul } A = \underline{2}$$

$$\dim \text{Row } A = \underline{5}$$

$$\text{If Col } A \text{ is a subspace of } \mathbb{R}^m, \text{ then } m = \underline{9}$$

$$\text{Rank } A = \underline{5}$$

$$\text{If Nul } A \text{ is a subspace of } \mathbb{R}^n, \text{ then } n = \underline{7}$$

2. If a basis for \mathbb{R}^3 is $B = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 0 \end{bmatrix} \right\}$, and given $[\vec{x}]_B = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, find \vec{x} in the standard basis.

$$P_B [\vec{x}]_B = \vec{x}$$

$$P_B = \begin{bmatrix} 1 & 5 & 4 \\ -2 & 0 & -3 \\ 3 & -2 & 0 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 1 & 5 & 4 \\ -2 & 0 & -3 \\ 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1-8 \\ -2+6 \\ 3+0 \end{bmatrix} = \begin{bmatrix} -7 \\ -4 \\ 3 \end{bmatrix}$$

3. If a vector in the standard basis is $\vec{x} = \begin{bmatrix} 1 \\ -7 \\ 8 \end{bmatrix}$, find its representation in the basis in problem #2.

$$P_B [\vec{x}]_B = \vec{x} \Rightarrow P_B^{-1} \vec{x} = [\vec{x}]_B$$

$$P_B^{-1} = \begin{bmatrix} 4/35 & 8/35 & 3/7 \\ 9/35 & 12/35 & 1/3 \\ -4/35 & -17/35 & -2/7 \end{bmatrix}$$

$$P_B^{-1} \vec{x} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}_B = [\vec{x}]_B$$

4. Consider the basis $C = \left\{ \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \right\}$, and the vector $[\vec{x}]_C = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$. Find the representation of the vector in the basis B in problem #2.

$$P_B [x]_B = P_C [x]_C = \vec{x}$$

$$[x]_B = P_B^{-1} P_C [x]_C$$

$$[x]_B = \begin{bmatrix} 6/35 & 8/35 & 3/7 \\ 9/35 & 12/35 & 1/3 \\ -4/35 & -17/35 & -2/7 \end{bmatrix} \begin{bmatrix} 1 & -3 & 2 \\ -1 & 4 & -2 \\ -3 & 9 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 606/35 \\ 244/35 \\ -509/35 \end{bmatrix}$$