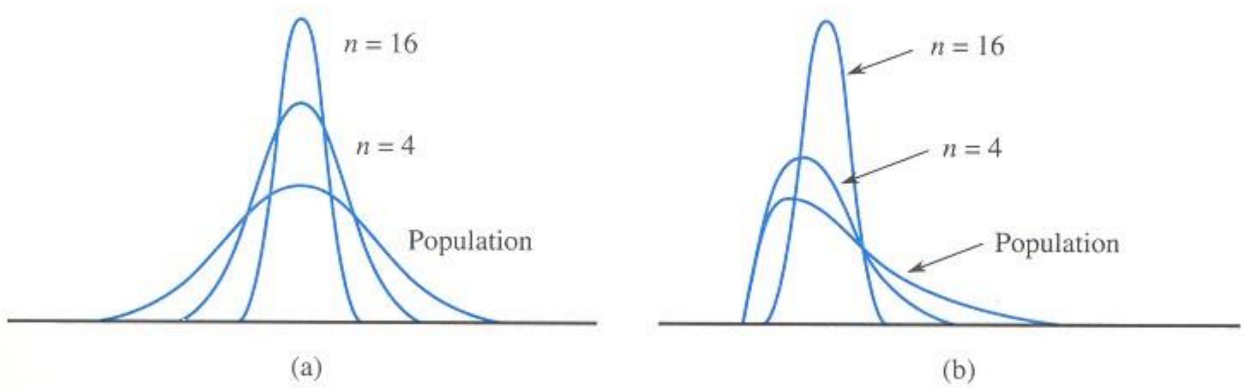
MTH 324, Homework #4, Fall 2022 Name ____

Instructions: Show work or attach R code used to perform calculations (or any other technology used). Be sure to answer all parts of each problem as completely as possible, and attach work to this cover sheet with a staple.

1. What does the Central Limit Theorem tell us about sampling distributions and why is it important?



2. Explain how these graphs relate to sampling distributions and the central limit theorem.

3. What is an unbiased estimator?

4. Rockwell hardness of pins of a certain type is known to have a mean value of 50 and a standard deviation of 1.2.
   a. If the distribution is normal, what is the probability that the same mean hardness for a certain random sample of 9 pins is at least 51?
   b. Without assuming population normality, what is the (approximate) probability that the sample of 40 pins is at least 51?

5. Below is a table with data for the flexural strength in MPa for a certain type of concrete beams.

5.9	**7.2**	**7.3**	**6.3**	**8.1**	**6.8**	**7.0**	**7.6**	**6.8**	**6.5**
7.0	6.3	7.9	9.0	8.2	8.7	7.8	9.7	7.4	7.7
9.7	7.8	7.7	11.6	11.3	11.8	10.7			

   a. Calculate a point estimate of the mean value of strength for the conceptual population of all beams of this type, and state which estimator you used and why.
   b. Calculate a point estimate of the strength value that separates the weakest 50% of all such beams from the strongest 50% and state which estimator you used.
   c. Calculate and interpret a point estimate of the population standard deviation σ. Which estimator did you use?
   d. Calculate a point estimate of the proportion of all such beams whose flexural strength exceeds 10 MPa.
   e. Calculate a point estimate of the population coefficient of variation $\frac{\sigma}{\mu}$, and state which estimator you used.

6. Each of 150 newly manufactured items is examined and the number of scratches per item is recorded (the items are supposed to be free of scratches), yielding the following data:

Number of scratches per item	0	1	2	3	4	5	6	7
Observed Frequency	18	37	42	30	13	7	2	1

Let X be the number of scratches on a randomly chosen item and assume that X has a Poisson distribution with parameter μ.

a. Find an unbiased estimator of μ and compute the estimate for the data. [Hint: $E(X) = \mu$ for X Poisson, so $E(\bar{X}) = ?$].

b. What is the standard deviation (standard error) of your estimator? Compute the estimated standard error. [Hint: $\sigma_X^2 = \mu$ for Poisson.]

7. For each of the following situations, find the maximum likelihood function. Then find the MLE for each parameter.

a. You find a die at your friend's house and think that it's coming up 4 entirely too frequently to be fair. You suspect it is weighted. To test this, you roll the die 25 times and obtain the following sequence of rolls: $\{4, 5, 2, 3, 4, 4, 1, 6, 4, 2, 3, 5, 4, 2, 6, 1, 4, 4, 1, 4, 2, 3, 5, 1, 6\}$. Use this information to find the maximum likelihood function to estimate the value of p= probability of obtaining a 4.

b. Suppose that SAT scores are distributed normally, and you'd like to calculate the mean and standard deviation upon which they are based. You obtain a sample of 10 scores for the quantitative section given by {510, 580, 430, 710, 220, 620, 550, 490, 700, 330}. Find the maximum likelihood function for μ, σ.

8. Suppose that a random sample of 50 bottles of a particular brand of cough syrup is selected and the alcohol content of each bottle is determined. Let μ denote the average alcohol content for the population of all bottles of the brand under study. Suppose the resulting 95% confidence interval is $(7.8, 9.4)$.

a. Would a 90% confidence interval calculated from this same sample have been narrower or wider than the given interval? Explain your reasoning.

b. Consider the following statement: There is a 95% chance that μ is between 7.8 and 9.4. Is this statement correct? Why or why not?

c. Consider the following statement: We can be highly confident that 95% of all bottles of this type of cough syrup have an alcohol content between 7.8 and 9.4. Is this statement correct? Why or why not?

d. Consider the following statement: If the process of selecting a sample of size 50 and then computing the corresponding 95% confidence interval is repeated 100 times, 95 of the resulting intervals will include μ. Is this statement correct? Why or why not?

9. By how much must the sample size n be increased if the width of the confidence interval is to be halved? If the sample size is increased by a factor of 25, what effect will this have on the width of the interval? Justify your assertions.

10. Determine the confidence level for each of the following large-sample, one-sided confidence bounds.

a. Upper bound: $\bar{x} + 0.83s/\sqrt{n}$
b. Lower bound: $\bar{x} - 2.05s/\sqrt{n}$
c. Upper bound: $\bar{x} + 0.67s/\sqrt{n}$

11. Determine the value of the following quantities.
    a. $t_{0.1,15}$ b. $t_{0.05,15}$ c. $t_{0.05,25}$ d. $t_{0.05,40}$ e. $t_{0.005,40}$

12. Determine the t critical value for a two-sided confidence interval in each of the following situations. And then describe the confidence interval upper and lower bound for each.
    a. Confidence level: 95%; df: 10
    b. Confidence level: 95%; df: 15
    c. Confidence level: 99%; df: 15
    d. Confidence level: 99%; df: 5
    e. Confidence level: 98%; df: 24
    f. Confidence level: 99%; df: 38

13. A sample of 14 joint specimens of a particular type gave a sample mean proportional limit stress of 8.48 MPa and a sample standard deviation of 0.79 MPa. Calculate and interpret a 95% confidence interval for the true average proportional limit stress of all such joints. What, if any, assumptions did you make about the distribution of proportional limit stress?

14. In a sample of 1000 randomly selected consumers who had opportunities to send in a rebate claim form after purchasing a product, 250 of these people said they never did so. Calculate an upper confidence bound at the 95% confidence level for the true proportion of such consumers who never apply for a rebate. Based on this bound, is there compelling evidence that the true proportion of such consumers is smaller than 1/3? Explain your reasoning.

15. What conditions need to be satisfied to use z-scores instead of t-scores?

16. What is the sample size for the 95% confidence interval $(5.8,7.2)$ if the standard deviation is known to be 1.9? What is the point estimate?