11/22/2022

Polar Coordinates (7.3)

https://www.geogebra.org/m/ApcfSCZY

Polar coordinates are based on the idea of locating a point in space based on the distance and the direction in form of an angle (relative to some reference).

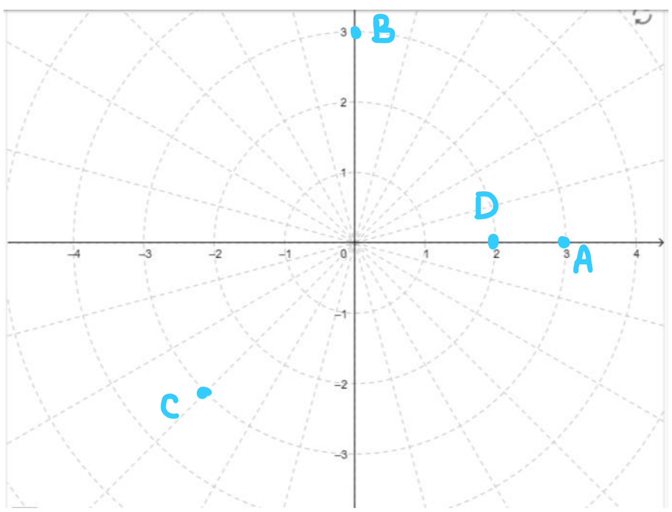
The distance from the origin is r (radius). The angle is measured from the positive x-axis, angles are measured in radians (this is because of calculus). Going counterclockwise.

The function variable in polar coordinates is r. So we write r as a function of theta: $r(\theta)$.

The coordinate point for polar coordinates is (r, θ).

Typically, r is positive. And theta is between 0 and 2π.

We can plot points with negative radii and angles outside the usual range. Points in the plane do not have unique representations.



Plotting points in polar coordinates. $A(3,0) = (-3,\pi), B\left(3,\frac{\pi}{2}\right) = \left(3,\frac{5\pi}{2}\right) = \left(-3,\frac{3\pi}{2}\right),$

$$C\left(3,\frac{5\pi}{4}\right), D(-2,-\pi) = (2,0) = (-2,\pi)$$

Converting points in rectangular coordinates to polar coordinates and/or polar coordinates to rectangular coordinates.

Conversion formulas:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$x^2 + y^2 = r^2$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \theta \rightarrow \tan\theta = \frac{y}{x}$$

Inverse tangent only produces angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. This covers angles only in the 1st or 4th quadrants. If you need 2nd or 3rd quadrants you can add π to the angle obtained from the formula (or, make the radius negative).

Points in rectangular and I want to convert to them to polar coordinates.

$$A(1,1)$$
$$B(0,2)$$
$$C(-2,5)$$

$$A(1,1), x = 1, y = 1, 1^2 + 1^2 = r^2 \rightarrow r = \sqrt{2}, \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$$
$$A\left(\sqrt{2}, \frac{\pi}{4}\right)$$

$$B(0,2), x = 0, y = 2, 0^2 + 2^2 = r^2 \rightarrow r = 2, \theta = \tan^{-1}\left(\frac{2}{0}\right) = \frac{\pi}{2}$$

When is inverse tangent undefined? $\frac{\pi}{2}$ or $-\frac{\pi}{2}$.

$$B\left(2, \frac{\pi}{2}\right)$$

$$C(-2,5), x = -2, y = 5, (-2)^2 + 5^2 = r^2 \rightarrow r = \sqrt{29}, \theta = \tan^{-1}\left(\frac{5}{-2}\right) \approx -1.190 + \pi \approx 1.951$$
$$C\left(\sqrt{29}, 1.951\right)$$

Convert points in polar coordinates to points in rectangular coordinates.

$$A\left(1, \frac{\pi}{6}\right)$$
$$B\left(4, -\frac{5\pi}{3}\right)$$
$$C(2,0)$$

$$A\left(1, \frac{\pi}{6}\right), r = 1, \theta = \frac{\pi}{6}, x = r\cos\theta = 1\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, y = r\sin\theta = 1\sin\frac{\pi}{6} = \frac{1}{2}$$
$$A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$
$$B\left(4, -\frac{5\pi}{3}\right), r = 4, \theta = -\frac{5\pi}{3}, x = 4\cos\left(-\frac{5\pi}{3}\right) = 4\left(\frac{1}{2}\right) = 2, y = 4\sin\left(-\frac{5\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$
$$B\left(2, 2\sqrt{3}\right)$$
$$C(2,0), r = 2, \theta = 0, x = 2\cos(0) = 2, y = 2\sin(0) = 0$$
$$C(2,0)$$

Converting formulas from rectangular to polar and/or polar to rectangular.

Rectangular to Polar

$$y = x^2$$

$$r\sin\theta = (r\cos\theta)^2$$
$$r\sin\theta = r^2\cos^2\theta$$

Divide out an r

$$\sin\theta = r\cos^2\theta$$

Since r is the function variable, solve for r if possible.

$$\frac{\sin\theta}{\cos^2\theta} = r$$
$$\tan\theta\sec\theta = r$$

This is an equation of a parabola in polar coordinates.

$$x^2 + y^2 = 9$$
$$r^2 = 9$$
$$r = 3$$

Convert from polar to rectangular functions.

$$r = 4\sin\theta$$

Multiply the equation by r on both sides

$$r^2 = 4r\sin\theta$$
$$x^2 + y^2 = 4y$$

$$x^2 + y^2 - 4y = 0$$

$$x^2 + y^2 - 4y + 4 = 4$$
$$x^2 + (y-2)^2 = 4$$

Circle with a center at (0,2) with a radius of 2.

Lines through the origin are very easy: θ = constant.

$$\theta = \frac{\pi}{4}$$

$$\tan\theta = \frac{y}{x} \to \tan\left(\frac{\pi}{4}\right) = \frac{y}{x} \to 1 = \frac{y}{x}$$

$$x = y$$

Graph polar functions.
See graphing program.
When we want to find the endpoints of the petals, set r=0.
On my archive site, I have a link at the bottom to pdfs of graph paper, that includes polar graph/polar graphing paper.