12/01/2022

Conic Sections (7.5)

Conic sections:
Shapes are derived from slicing through a cone at different angles.

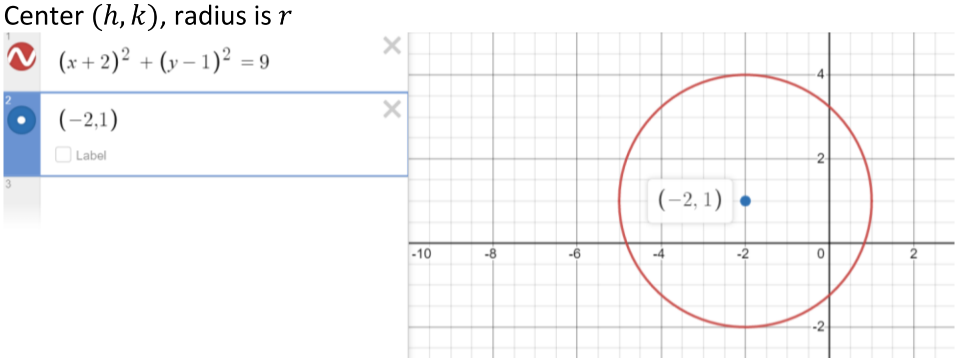
Circle
Ellipse
Parabola
Hyperbola

Focus (foci), center, directrix, asymptotic lines (hyperbolas), eccentricity

Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

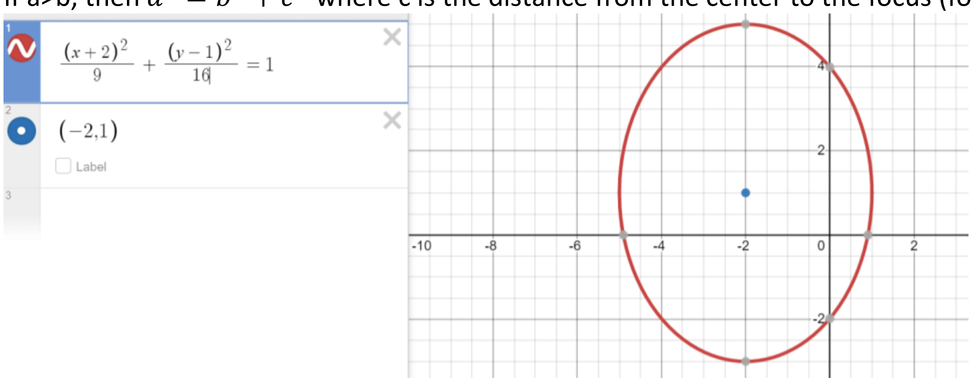
Center (h, k), radius is r



Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

If a>b, a is the length of the semi-major axis, and b is the length of the semi-minor axis
(h, k) is the center.
If a>b, then $a^2 = b^2 + c^2$ where c is the distance from the center to the focus (foci)



Parabola

$$y - k = \frac{1}{4p}(x-h)^2$$

The point (h, k) is the vertex.

p is the distance from the vertex to the focus $(h, k + p)$, and in the opposite direction, to the directrix $y = k - p$ (line).

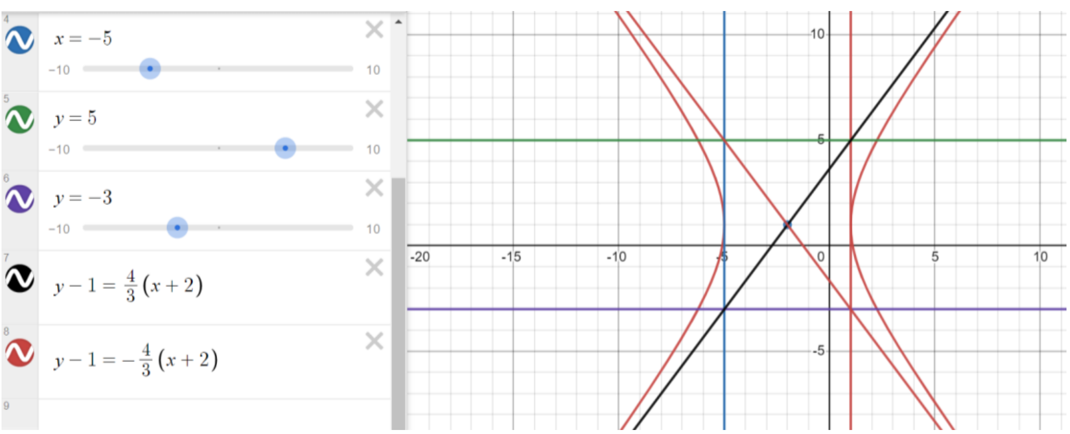
$$x - h = \frac{1}{4p}(y - k)^2$$

The point (h, k) is the vertex.
p is the distance from the vertex to the focus $(h + p, k)$, and in the opposite direction, to the directrix $x = h - p$ (line).

Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

The point (h, k) is the center. There is no relationship between a and b in terms of size. $a^2 + b^2 = c^2$ where c is the distance from the center to foci, and a (on the positive term) is the distance from the center to the vertices.
Use b to find the "box" that makes the asymptotes along with a.



Conic Sections in Polar Coordinates.

Eccentricity

$$e = \frac{c}{a}$$

The ratio of the distance to the focus over the distance to the vertex.
$e = 0$ is the eccentricity of a circle
$0 < e < 1$ is the eccentricity of an ellipse
$e = 1$ the eccentricity of a parabola

$e > 1$ is the eccentricity of a hyperbola

General form for conic sections in polar coordinates is:

$$r = \frac{ep}{1 \pm e\cos\theta}$$

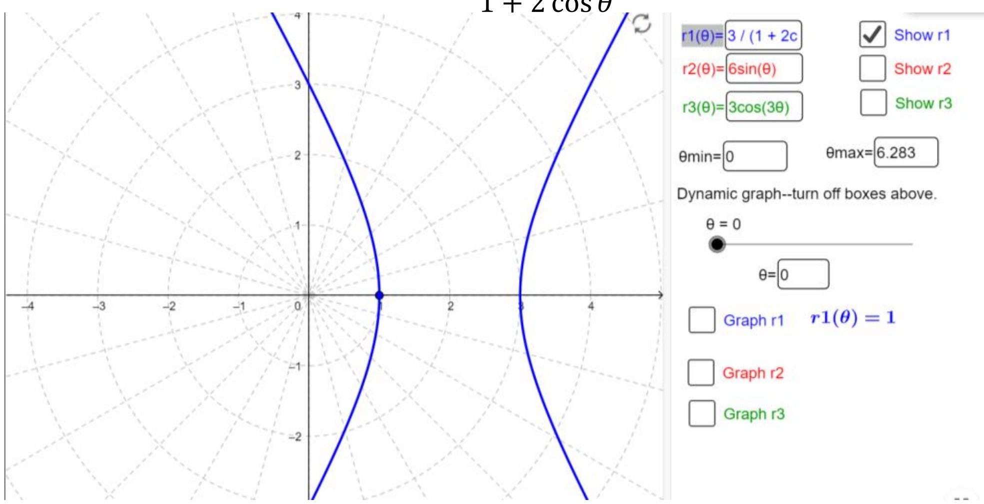
$$r = \frac{ep}{1 \pm e\sin\theta}$$

e is eccentricity, p is focal parameter, the trig functions determine whether the graph is oriented LR or UD, with the sign determining left vs. right, or up vs. down.

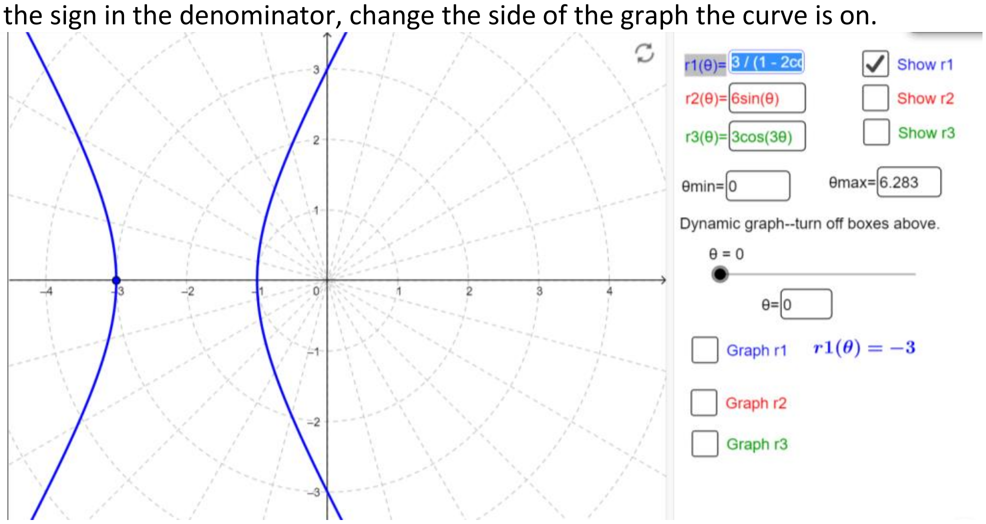
Circles are special cases. When centered at the origin they are just $r = constant$. When you shift it off the origin but so that it still passes through the origin, $r = a\sin\theta$ or $r = a\cos\theta$.

Example.

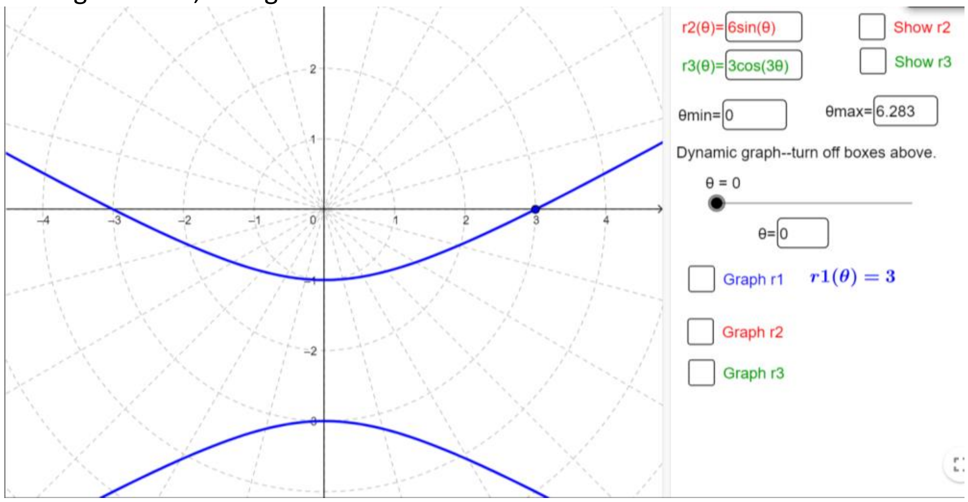
$$r = \frac{3}{1 + 2\cos\theta}$$



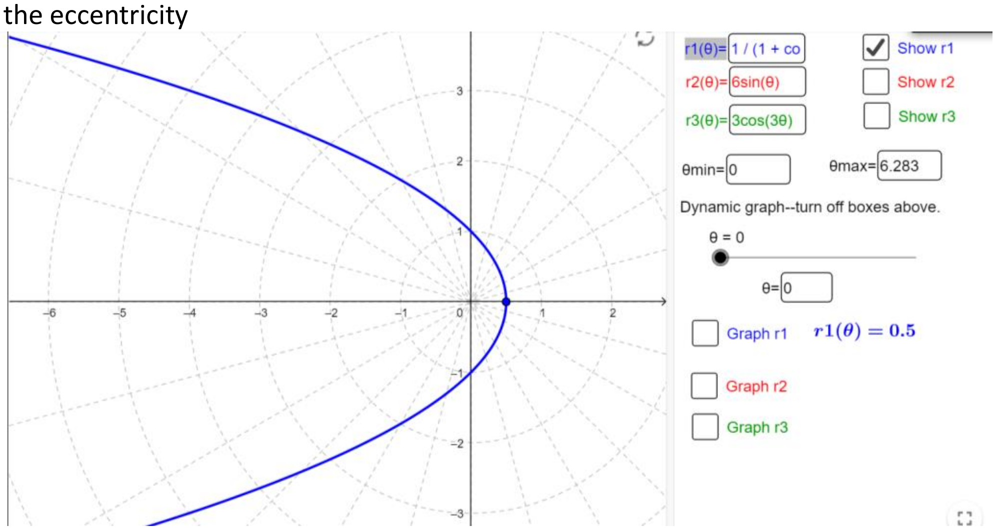
Change the sign in the denominator, change the side of the graph the curve is on.

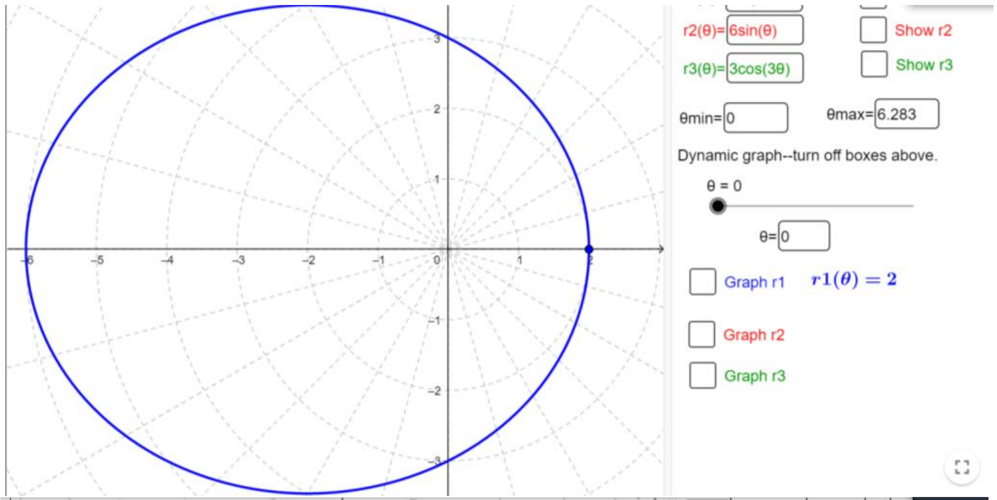


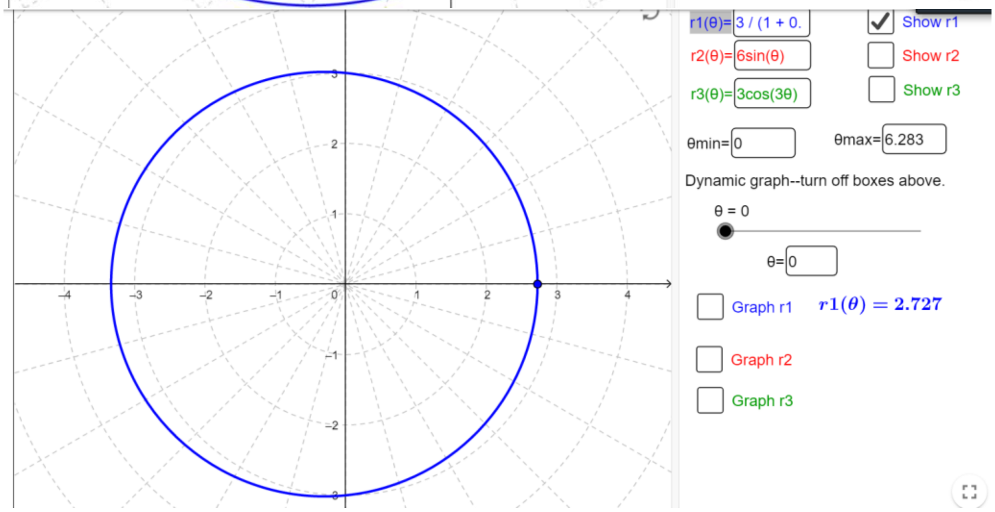
Change the trig function, change the orientation



Change the eccentricity







Suppose we have the function $r = \frac{3}{2+\cos\theta}$. Identify the type of conic section.
Match the form $r = \frac{k}{1+e\cos\theta}$

$$r = \frac{3}{2+\cos\theta} \times \frac{\left(\frac{1}{2}\right)}{\frac{1}{2}} = \frac{\left(\frac{3}{2}\right)}{1+\frac{1}{2}\cos\theta}$$

$$e = \frac{1}{2}$$

General form of a conic in rectangular coordinates

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

If A=C, this is a circle
If A,C>0 (but not equal), then it's an ellipse
If A and C are different signs, then it's a hyperbola

And if only A or C are in the equation, then it's a parabola.

If you need to know anything else about the equation, you'll need to complete the square for x and y.

xy-term is for rotated conics which we won't we working with.

The last lecture!!!
Dec 6 meeting is technically optional. It's only to do review for the final.
If you plan to attend, please bring questions.
The final is the same format as previous exams, on Dec 8th.
There is a quiz due tonight, and the last one is due Dec 6th.