MTH 264, Quiz #11, Fall 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the first 4 non-zero terms for the Taylor polynomial that approximates $y = \sinh(x)$. [Note: there is at least two ways to do this that we have learned. You may use either method.]

$c=0$

n	$n!$	$f^{(n)}(x)$	$f^{(n)}$	$(x-c)^n$	$f^{(n)}(c)(x-c)^n/n!$
0	1	$\sinh x$	0	1	0
1	1	$\cosh x$	1	x	x
2	2	$\sinh x$	0	x^2	0
3	6	$\cosh x$	1	x^3	$x^3/6$
4	24	$\sinh x$	0	x^4	0
5	120	$\cosh x$	1	x^5	$x^5/120$
6	720	$\sinh x$	0	x^6	0
7	5040	$\cosh x$	1	x^7	$x^7/5040$

$$\sinh x = x + \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^7}{5040} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

Same as $\sin x$ except not alternating signs

2. Given that the Taylor series for $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, use this fact to write the function e^{-x^2} as a power series and find an expression for the integral $\int e^{-x^2}\,dx$.

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

$$\int e^{-x^2}\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}\,dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int x^{2n}\,dx =$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n+1}}{(2n+1)}$$