

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Find the slope of the tangent line to the graph $r = 4 - \cos \theta$ when $\theta = \frac{\pi}{4}$. You may use the formula $\frac{dy}{dx} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$. [Hint: to write the line, you may do it in rectangular coordinates, but you will have to convert the point on the polar graph to rectangular as well.]

$$f'(x) = +\sin \theta$$

$$\frac{dy}{dx} = \frac{\sin \theta (\sin \theta) + (4 - \cos \theta) \cos \theta}{\sin \theta \cos \theta - (4 - \cos \theta) \sin \theta} = \frac{\sin^2 \theta - 4 \cos \theta - \cos^2 \theta}{2 \sin \theta \cos \theta - 4 \sin \theta} =$$

$$\frac{\left(\frac{1}{\sqrt{2}}\right)^2 - 4\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)^2}{2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) - 4\left(\frac{1}{\sqrt{2}}\right)} = \frac{-4/\sqrt{2}}{1 - 4/\sqrt{2}}$$

2. Find the area of one petal of the graph $r = 2 \cos 2\theta$. Sketch the graph.

$$\frac{1}{2} \int_{-\pi/4}^{\pi/4} (2 \cos 2\theta)^2 d\theta =$$

$$\text{by symmetry } \frac{1}{2} \cdot 2 \int_0^{\pi/4} (2 \cos 2\theta)^2 d\theta$$

$$= 4 \int_0^{\pi/4} \cos^2 2\theta d\theta = 4 \int_0^{\pi/4} \frac{1}{2} (1 + \cos 4\theta) d\theta =$$

$$2 \left[\theta + \frac{1}{4} \sin 4\theta \right]_0^{\pi/4} = 2 \left(\frac{\pi}{4} \right) = \frac{\pi}{2}$$

