

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Determine if the integral converges. If it does, state the value. If it does not, show work to explain why not.

$$\int_0^{\infty} x e^{-x^2} dx$$

$$u = -x^2$$

$$du = -2x dx$$

$$-\frac{1}{2} du = x dx$$

$$\int -\frac{1}{2} e^u du =$$

$$-\frac{1}{2} e^u \rightarrow -\frac{1}{2} e^{-x^2} \Big|_0^{\infty}$$

$$\lim_{b \rightarrow \infty} -\frac{1}{2} e^{-b^2} + \frac{1}{2} e^0 = \frac{1}{2} \quad \text{Converges}$$

2. For the following integrals, state which method you would use, and which basic integration rule. Do not actually perform the integration. Methods may include: substitution, change of variables, complete the square, add/subtract, trig identities, long division, partial fractions, by parts, trig substitution, etc. Basic integration rules may include: power rule, log rule, exponential rule, trig functions, inverse trig functions, etc. Some problems may require more than one method or rule.

a. $\int \frac{x^2}{4x^2 + 9} dx$

METHOD
long division

RULE
arctan + power rule

b. $\int \frac{1}{x^2 + 6x - 7} dx$

METHOD
factor, partial fractions

RULE
log rule

c. $\int \frac{x}{x^4 + 2x^2 + 2} dx$

METHOD
complete the square, substitution, arctan

RULE

d. $\int \frac{x}{\sqrt{x-2}} dx$

METHOD
change of variables or
integration by parts

RULE

power rule