MTH 264, Quiz #8, Fall 2022 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Use the integral test or the p-series test to determine if the series converges. State which one you used.

   a. $\sum_{n=1}^{\infty} \frac{3}{n^{5/3}}$ p-series test $p>1$ since $p=5/3$

   Converges

   b. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln n}}$ integral test $u=\ln n$, $du=\frac{1}{n}\,dn$ $\int \frac{1}{n\sqrt{\ln n}}\,dn = \int u^{-1/2}\,du =$

   $2\cdot u^{1/2} \to 2\sqrt{\ln n}$

   $\int_1^{\infty} \frac{1}{n\sqrt{\ln n}}\,dn = 2\sqrt{\ln n}\Big|_1^{\infty} = \lim_{b\to\infty} 2\sqrt{\ln b} - 0 = \infty$ diverges

2. Use the direct comparison test or the limit comparison test to determine if the series converges or diverges. State which one you used.

   a. $\sum_{n=1}^{\infty} \frac{3^n}{4^n+5}$ compare w/ geometric $\left(\frac{3}{4}\right)^n$ limit comparison

   $\lim_{n\to\infty} \frac{\cancel{3^n}}{4^n+5}\cdot\frac{4^n}{\cancel{3^n}} = \lim_{n\to\infty} \frac{4^n}{4^n+5} = 1$ converge/diverge together

   $(3/4)^n$ converges, so this also converges

   can also apply direct comparison since $\frac{3^n}{4^n+5} \le \left(\frac{3}{4}\right)^n$

   b. $\sum_{n=1}^{\infty} \frac{1}{n(n^2+1)}$

   compare w/ $\frac{1}{n^3}$

   $\lim_{n\to\infty} \frac{1}{n(n^2+1)}\cdot\frac{n^3}{1} = \lim_{n\to\infty} \frac{n^{\cancel{3}}}{\cancel{n}(n^2+1)} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

   converge or diverge together. $\frac{1}{n^3}$ converges by the p-test

   so this converges

   can also use direct comparison since $\frac{1}{n(n^2+1)} < \frac{1}{n^3}$