MTH 267, Homework #2, Fall 2022 Name ____________________________________

Instructions: Work problems on a separate sheet of paper and attach work to this page. You should show all work to receive full credit for problems. Checking your work with computer algebra systems is fine, but that doesn't count as "work" since you won't be able to use CAS programs on exams or quizzes. Graphs and longer answers that won't fit here, indicate which page of the work the answer can be found on and be sure to clearly indicate it on the attached pages.

1. Use Euler's method to approximate the solution at the specified point using the requested number of steps, or the given step size. You should do at least three steps by hand (for any problem with 3 or more steps), but the remainder you can compute in Excel or another similar program.
   a. $\frac{dy}{dt} = 2y - 1, y_0(1) = 0, \Delta t = 0.5, y(2) = ?$
   b. $\frac{dy}{dt} = 5 - 3\sqrt{y}, y_0(1) = 4, \Delta t = 0.1, y(1.5) = ?$
   c. $\frac{dy}{dt} = y(2 - ty), y_0(2) = 1, n = 3, y(3) = ?$
   d. $\frac{dy}{dt} = \frac{3t^2}{y^2-4}, y_0(3) = 1, n = 20, y(7) = ?$

2. Consider the initial value problems given below. Use Euler's method to approximate the solutions at t=0.5, using h=0.1, h=0.05, h=0.01. You may wish to set up an Excel program to do the calculation for h=0.01. (You should do at least the case for h=0.1 by hand.) If the equation can be solved exactly by some method we've learned so far, compare your approximations to the true value. Report your approximations with at least 4 decimal places (carry 6 places through your calculations).
   a. $y' = 2y - 1, y(0) = 1$
   b. $y' = y(3 - ty), y(0) = 2$

3. Solve the following differential equations by separation of variables. If an initial value is provided, be sure to find all unknown constants. If the solution is not valid for all values of (t,y) state the intervals where it is valid.
   c. $y' = \frac{x^2}{y(1+x^3)^4}$
   c. $y' + y^2 \sin x = 0$
   d. $xy' = (1 - y^2)^{\frac{1}{2}}$
   d. $y' = \frac{1-2x}{y}, y(1) = -2$
   e. $\sin 2x\, dx + \cos 3y\, dy = 0, y\left(\frac{\pi}{2}\right) = \frac{\pi}{3}$

4. Suppose that a certain population has growth rate that varies with time and that this population satisfies the differential equation $\frac{dy}{dt} = \frac{(0.5+\sin t)y}{5}$. If $y(0) = 1$, find (or estimate) the time τ at which the population has doubled. Choose other initial conditions and determine whether the doubling rate depends on the initial population.

5. Determine the interval on which each differential equation (together with its initial value) has a unique solution. [Hint: if $y' = f(t, y)$ check for continuity in both f and $\frac{\partial f}{\partial y}$.] It's possible that the solution will be a region in the ty-plane rather than an interval. In such a case, provide a graph of the region.
   a. $(t - 3)y' + (\ln t)y = 2t,\ y(1) = 2$
   b. $y' = (1 - t^2 - y^2)^{1/2}$
   c. $(4 - t^2)y' + 2ty = 3t^2, y(-3) = 1$
   d. $\frac{dy}{dt} = \frac{(\cot t)y}{1+y}$

6. Determine the intervals on which the solutions are sure to exist.

   e. $y^{IV} + 4y''' + 3y = t$ $\qquad$ $y''' + ty'' + t^2y' + t^3y = \ln t$