MTH 267, Quiz #3, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the differential equation $\frac{dy}{dt} = 4 + y$ for the analytic solution. Solve for the missing constant if the initial condition is y(0)=1. (Use linear/integrating factor methods.)

$y' - y = 4 \qquad \mu = e^{\int -dt} = e^{-t}$

$e^{-t}y' - e^{-t}y = 4e^{-t}$

$\int (e^{-t}y)' = \int 4e^{-t}$

$e^{-t}y = -4e^{-t} + C$

$y = -4 + Ce^{t}$

$1 = -4 + C$

$C = 5$

$y(t) = 5e^{t} - 4$

2. Solve the differential equation $y' = \frac{x^2 - y^2}{xy}$.

$y = vx \qquad y' = xv' + v \qquad v = \frac{y}{x}$

$xv' + v = \frac{x^2 - v^2x^2}{xvx} = \frac{\cancel{x^2}(1-v^2)}{\cancel{x^2}v}$

$xv' + v = \frac{(1-v^2)}{v} = \frac{1}{v} - v$
$\quad -v \qquad\qquad\qquad\quad -v$

$xv' = \frac{1}{v} - 2v = \frac{1-2v^2}{v}$

$\int \frac{dv \cdot v}{1-2v^2} = \int \frac{1}{x}\,dx$

$u = 1 - 2v^2 \Rightarrow$

$du = -4v\,dv$

$-\frac{1}{4}du = v\,dv$

$\int -\frac{1}{4} \cdot \frac{1}{u}\,du = \int \frac{1}{x}\,dx$

$-\frac{1}{4}\ln u = \ln x + C$

$u^{-1/4} = Ax$

$u = \frac{1}{\cancel{B}x^4} = \frac{C}{x^4}$

$1 - 2v^2 = \frac{C}{x^4}$

$\boxed{1 - 2\left(\frac{y^2}{x^2}\right) = \frac{C}{x^4}}$

3. Solve the Bernoulli equation $y' + \frac{3}{x}y = \frac{4}{x}e^{-2x}y^{\frac{4}{3}}, y(1) = 2$.

$-\frac{1}{3}y^{-4/3}y' + \left(-\frac{1}{3}\cdot\frac{3}{x}\right)y^{-4/3}y = -\frac{1}{3}\left(\frac{4}{x}e^{-2x}\right)$

$-\frac{1}{3}y^{-4/3}y' - \frac{1}{x}y^{-1/3} = -\frac{4}{3x}e^{-2x}$

$z' - \frac{1}{x}z = -\frac{4}{3x}e^{-2x}$

$\frac{1}{x}z' - \frac{1}{x^2}z = -\frac{4}{3x^2}e^{-2x}$

$\int \left(\frac{1}{x}z\right)' = \int \frac{4}{3x^2}e^{-2x}$

$\frac{1}{x}z = \int \frac{4}{3x^2}e^{-2x}\,dx$

$n = 4/3$

$(1 - \frac{4}{3})y^{-4/3}$

$-\frac{1}{3}y^{-4/3}$

$z = y^{-1/3}$

$z' = -\frac{1}{3}y^{-4/3}y'$

$\mu = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = e^{\ln(\frac{1}{x})} = \frac{1}{x}$

$y^{-1/3} = x\int_{1}^{x} \frac{4}{3t^2}e^{-2t}\,dt + Cx$

$2^{-1/3} = 1\int_{1}^{1} \frac{4}{\ldots} e^{-2t}\,dt + C(1) \Rightarrow C = 2^{-1/3}$

$\boxed{y^{-1/3} = x\int_{1}^{x} \frac{4}{3t^2}e^{-2t}\,dt + 2^{-1/3}}$