MTH 267, Quiz #7, Fall 2022 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use reduction of order to solve $t^2y'' - t(t+2)y' + (t+2)y = 0, y_1 = t$.

$y_2 = vy_1 = vt$

$y_2' = v't + v$

$y_2'' = v''t + 2v'$

$t^2(v''t + 2v') - \overset{(t^2+2t)}{t(t+2)}(v't + v) + (t+2)vt = 0$

$t^3v'' + \cancel{2t^2v'} - \cancel{t^3v'} - \cancel{t^2v} - \cancel{2t^2v'} - \cancel{2tv} + \cancel{t^2v} + \cancel{2tv} = 0$

$t^3v'' - t^3v' = 0$

$t^3(v'' - v') = 0$

$u = v'$

$u' = v''$

$u' - u = 0$

$\frac{du}{dt} = u$

$\int \frac{du}{u} = \int dt$

$\ln u = t + c$

$u = Ae^t$

$v' = Ae^t$

$v = \int Ae^t\,dt = Ae^t + c$

or Ae^t

$y_1 = t$

$y_2 = Ae^t \cdot t = Ate^t$

$\boxed{y(t) = c_1 t + c_2 te^t}$

2. Solve the homogeneous higher order equation $t^3y''' - 3ty' + y = 0$.

$y = x^r = t^r$

$y' = rx^{r-1} = rt^{r-1}$

$y'' = rx^{r-2}(r-1) = r(r-1)t^{r-2}$

$y''' = r(r-1)(r-2)x^{r-3} = r(r-1)(r-2)t^{r-3}$

$t^3 r(r-1)(r-2)t^{r-3} - 3tr t^{r-1} + t^r = 0$

$r(r-1)(r-2)t^r - 3rt^r + t^r = 0$

$t^r[r(r-1)(r-2) - 3r + 1] = 0$

$(r^2 - r)(r-2) - 3r + 1 = 0$

$r^3 - 2r^2 - r^2 + 2r - 3r + 1 = 0$

$r^3 - 3r^2 - r + 1 = 0$ not factorable/real solutions

$y(t) \approx c_1 t^{-0.675} + c_2 t^{0.461} + c_3 t^{3.214}$