11/21/2023

Polar Coordinates

Conversion formulas
Converting between rectangular and polar (and vice versa) for points and equations
Graphing polar curves
Converting polar equations into parametric form
Symmetry

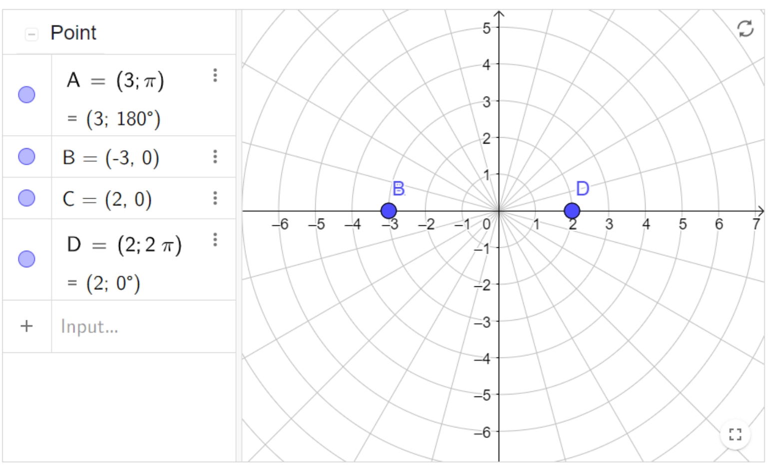
Polar coordinates depend on an angle relative to the positive x-axis. Angles go counterclockwise. The second coordinate is the distance from the origin.

Coordinate pairs are (r, θ).

By default, we assume r is positive, but in functions (equations) r can be negative.

Polar coordinates are not unique. In rectangular coordinates, every point in the plane has a unique representation. But not in polar. To get another representation of the same point, add 2π to the angle (or subtract it), or make the radius negative and add π to the angle.

https://www.geogebra.org/m/vwv8acug



Switch between rectangular and polar and back again

Identities for converting coordinate systems:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$x^2 + y^2 = r^2$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Converting rectangular coordinate points to polar.
Convert the point $(1, -\sqrt{3})$ to polar.

$$r^2 = (1)^2 + \left(-\sqrt{3}\right)^2 = 4, r = 2$$
$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) = -\frac{\pi}{3}$$

$$\left(2, -\frac{\pi}{3}\right), \left(-2, \frac{4\pi}{3}\right)$$

Converting polar coordinates to rectangular coordinates.
Convert the point $\left(4, \frac{2\pi}{3}\right)$ to rectangular.

$$x = 4\cos\left(\frac{2\pi}{3}\right) = 4\left(-\frac{1}{2}\right) = -2, y = 4\sin\left(\frac{2\pi}{3}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$$

$$\left(-2, 2\sqrt{3}\right)$$

Converting from rectangular to polar (or vice versa) in function/equation form.

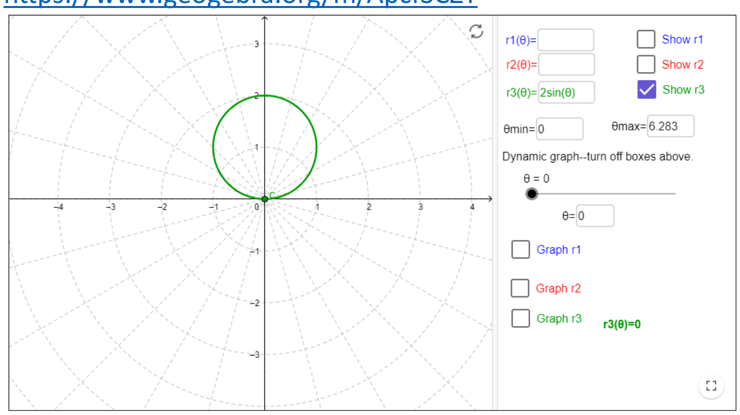
$$x^2 + y^2 = 2y$$

$$r^2 = 2r\sin\theta$$

Divide out the common r

$$r = 2\sin\theta$$

This is your polar equation.
https://www.geogebra.org/m/ApcfSCZY



$$x^2 + y^2 - 2y = 0$$
$$x^2 + y^2 - 2y + 1 = 1$$

$$x^2 + (y-1)^2 = 1$$

Circle, centered at (0,1) with a radius of 1.

Suppose we have the equation $r = 4\cos\theta$.
Don't use the square root of the circle equation.

The trick is to multiply both side of the equation by r

$$r^2 = 4r\cos\theta$$

$$x^2 + y^2 = 4x$$

The important thing is to make sure that you eliminate all cases of r and theta. And you need to simplify any trig/inverse trig expressions.

Don’t do this:

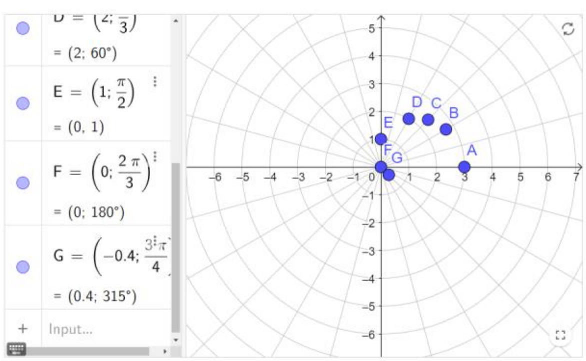
$$\sqrt{x^2+y^2} = 4\cos\left(\tan^{-1}\frac{y}{x}\right)$$

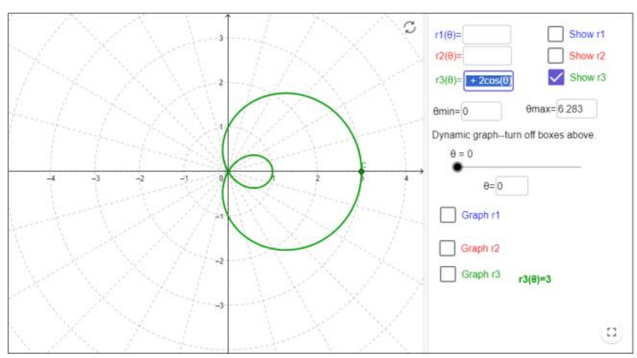
This is terrible.

Graphing polar curves.

$$r = 1 + 2\cos\theta$$

θ	r
0	$1 + 2(1) = 3$
$\frac{\pi}{6}$	$1 + 2\left(\frac{\sqrt{3}}{2}\right) = 1 + \sqrt{3} \approx 2.7$
$\frac{\pi}{4}$	$1 + 2\left(\frac{\sqrt{2}}{2}\right) = 1 + \sqrt{2} \approx 2.4$
$\frac{\pi}{3}$	$1 + 2\left(\frac{1}{2}\right) = 2$
$\frac{\pi}{2}$	$1 + 2(0) = 1$
$\frac{2\pi}{3}$	$1 + 2\left(-\frac{1}{2}\right) = 1 - 1 = 0$
$\frac{3\pi}{4}$	$1 + 2\left(-\frac{\sqrt{2}}{2}\right) = 1 - \sqrt{2} \approx -0.4$





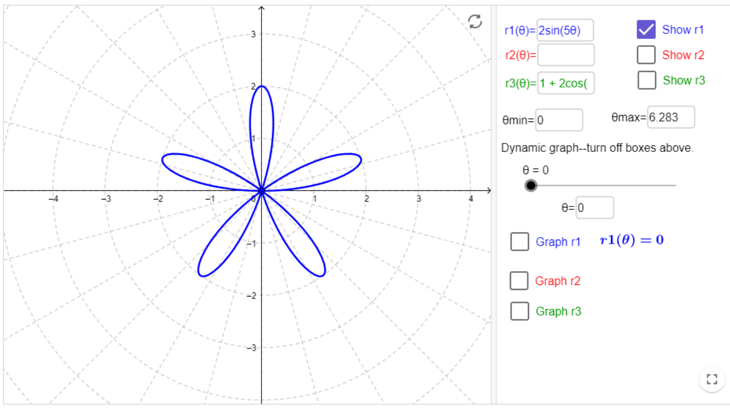
Symmetry

Symmetry with respect to the x-axis and symmetry with respect to the y-axis.

See above for an example with symmetry to the x-axis.

If you plot the entire graph for angles 0 through pi, the rest of the graph will be the mirror image of those same points. This typically occurs for “even” symmetry, so cosine functions will be symmetric to the x-axis,

If there is y-axis symmetry, this typically occurs with “odd” functions, so sine. Plot the angles between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, then the second and third quadrants would be the mirror points.



Polar graphs in parametric form.

What we use are the $x = r\cos\theta$, and $y = r\sin\theta$

Suppose we want to convert $r = 3$ into parametric form.

The parametric form is $x(t) = 3\cos t\,, y(t) = 3\sin t$

What if I wanted to put $r = 2\sin(5\theta)$ into parametric form?

$$x(t) = 2\sin(5t)\cos t\,, y(t) = 2\sin(5t)\sin t$$