MTH 264, Quiz #11, Fall 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Use the table of Taylor series/power functions to find power series expressions for the following functions. Develop a general formula centered at c=0.

   a. $f(x) = \cos x^{3/2}$

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n \left(x^{3/2}\right)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{(2n)!}$$

   b. $f(x) = \cos^2 x$

$$\left(1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{\underset{24}{4!}} - \frac{x^6}{6!} + \cdots\right)$$

$$1 - \frac{x^2}{2} + \frac{x^4}{\underset{24}{4!}} - \frac{x^6}{\underset{720}{6!}} - \frac{x^2}{2} + \frac{x^4}{4} - \frac{x^6}{48} \cdots + \frac{x^4}{24} - \frac{x^6}{24^2} - \frac{x^6}{720} \cdots$$

$$1 - x^2 + \frac{x^4}{3} - \frac{13}{576}x^6 + \cdots$$

2. Use a power series to evaluate $\lim_{x \to 0} \frac{\ln(x+1)}{x}$.

$$\lim_{x \to 0} \frac{x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots}{x} = \lim_{x \to 0} 1 + \frac{1}{2}x + \frac{1}{3}x^2 + \frac{1}{4}x^3 \cdots = 1$$

3. Integrate $\int_0^1 e^{-x^3}\,dx$ using a Taylor series using 5 terms of the series.

$$e^{-x^3} = \sum_{n=0}^{\infty} \frac{(x^3)^n(-1)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{n!}$$

$$\sum \int \frac{(-1)^n x^{3n}}{n!}\,dx = \sum \frac{(-1)^n x^{3n+1}}{(3n+1)\,n!}\Bigg|_0^1$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 1}{(3n+1)n!} - 0 \approx \frac{(-1)^0(1)}{(1)(1)} + \frac{(-1)^1(1)}{(4)(1)} + \frac{(-1)^2(1)}{7(2)} + \frac{(-1)^3(1)}{10(6)} + \frac{(-1)^4(1)}{13(24)}$$

$$1 + \left(\frac{-1}{4}\right) + \frac{1}{14} - \frac{1}{[illegible]} + \frac{1}{[illegible]} \approx \underline{2941}$$