MTH 264, Quiz #5, Fall 2023 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

1. Integrate using the integrals table posted in Canvas. Note which formula you used.

   a. $\int \frac{\ln x}{x(3+2\ln x)}dx$

   $u = \ln x \quad du = \frac{1}{x}dx$

   $\int \frac{u}{3+2u}du \qquad 2.6 \quad \int \frac{x}{ax+b}dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$

   $= \frac{u}{2} - \frac{3}{4}\ln|2u+3| + C = \frac{\ln x}{2} - \frac{3}{4}\ln|2\ln x+3| + C$

   b. $\int \frac{x^2}{(2x-7)^{\frac{3}{2}}}dx \qquad a=2 \; b=-7$

   $3.6 \quad \int x^m (ax+b)^{(2n+1)/2}dx = \frac{2}{a(2n+3)}\left[x^m (ax+b)^{\frac{(2n+3)}{2}} - m\int x^{m-1}(ax+b)^{\frac{2n+3}{2}}\right] \qquad m=2, \; n=-1$

   $= \frac{2}{2(1)}\left[x^2(2x-7)^{-1/2} - 2\int x(2x-7)^{-1/2}dx\right] + C$

   $3.7 \quad \int \frac{x}{\sqrt{ax+b}}dx = \frac{2x}{a}\sqrt{ax+b} - \frac{4b}{3a^2}(ax+b)^{3/2} + C$

   $-x^2(2x-7)^{-1/2} + 2\left[\frac{2x}{2}(2x-7)^{1/2} + \frac{28}{12}(2x-7)^{3/2}\right] + C$

   $\frac{-x^2}{\sqrt{2x-7}} + 2x\sqrt{2x-7} + \frac{14}{3}(2x-7)^{3/2} + C$

2. Use the indicated method and number of subintervals to estimate the value of $\int_0^2 \sqrt{1+x}\,dx$.

   a. Trapezoidal Rule, n=6

   $\Delta x = \frac{2-0}{6} = \frac{1}{3}$

   $\frac{1}{6}\left[f(0) + 2f(\tfrac{1}{3}) + 2f(\tfrac{2}{3}) + 2f(1) + 2f(\tfrac{4}{3}) + 2f(\tfrac{5}{3}) + f(2)\right] \approx 2.795484$

   b. Simpson's Rule, n=4

   $\Delta x = \frac{2}{4} = \frac{1}{2}$

   $\frac{1}{6}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right] = 2.797335$

   c. What is the estimated error in each case (use the error formula, not the exact value of the integral)?

   $f'(x) = \frac{1}{2}(x+1)^{-1/2}$

   $f''(x) = -\frac{1}{4}(x+1)^{-3/2} \quad \max\left|-\frac{1}{4}(1)^{-3/2}\right| = \frac{1}{4}$

   $f'''(x) = \frac{3}{8}(x+1)^{-5/2}$

   $f^{IV}(x) = -\frac{15}{16}(x+1)^{-7/2} \quad \max\left|-\frac{15}{16}(1)^{-7/2}\right| = \frac{15}{16}$

   trap. $\frac{1/4(2-0)^3}{12(6)^2} = \frac{2}{432} = \frac{1}{216}$

   Simp $\frac{15/16(2-0)^5}{180(4)^4} = \frac{\frac{15}{16}\cdot 32}{180(4)^4} = \frac{30}{46080} \approx 6.51 \times 10^{-4}$