MTH 161, Exam #2, Fall 2024 Name KEY

Instructions: Work the problems below as directed. Show all work. Clearly mark your final answers. Use exact values unless the problem specifically directs you to round. Simplify as much as possible. Partial credit is possible, but solutions without work will not receive full credit.

Part 1: These questions you will submit answers to in Canvas. Show all work and submit the work with Part 2 of the exam. But you must submit the answers in Canvas to receive credit. Each question/answer will be listed separately. The Canvas question will refer to the number/part to indicate where you should submit which answer. The questions will appear in order (in case there is an inadvertent typo). Correct answers will receive full credit with or without work in this section, but if you don't submit work and clearly label your answers, you won't be able to challenge any scoring decisions for making an error of any kind.

1. Simplify the following expressions. Write each in standard form. (10 points each)
   a. $8 - (-7 + 3i) - (-14 - 4i)$

$$8+7-3i+14+4i = 15+14-3i+4i = 29+i$$

   b. $(2 - 5i)^2$

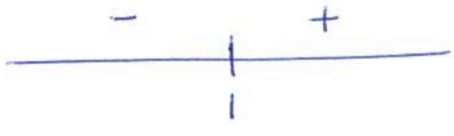
$$4-20i+25\underset{\searrow(-1)}{i^2} = 4-20i-25 = -21-20i$$

   c. $\frac{1+4i}{2-i}$ $\cdot\frac{2+i}{2+i} = \frac{2+i+8i+9\overset{(-1)\swarrow}{i^2}}{4+1} = \frac{2+9i-4}{5} = \frac{-2+9i}{5} = -\frac{2}{5}+\frac{9i}{5}$

2. Solve the rational and polynomial inequalities and write the solution in interval notation. Be sure to create a sign chart. You should consider checking your answer with a graph. (16 points each)
   a. $x^3 - x^2 + 4x - 4 > 0$

$$x^2(x-1)+4(x-1)>0$$

$$\underset{\neq 0 \nearrow}{(x^2+4)}(x-1)>0$$

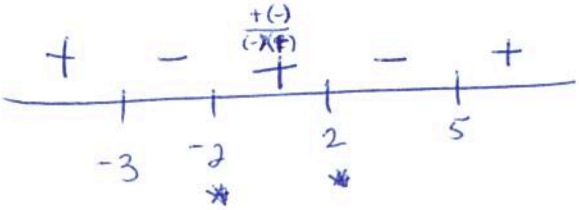


$(1, \infty)$

b. $\frac{(x+3)(x-5)}{x^2-4} \le 0$

$\frac{(x+3)(x-5)}{(x-2)(x+2)} \le 0$

$x = 2, -2, -3, 5$



$[-3, -2) \cup (2, 5]$

3. Find all the possible rational zeros of the polynomial $f(x) = 3x^3 - 13x^2 + 43x - 13$. Use them to factor the polynomial, and find all the real (and complex, if any) zeros. Write the polynomial in factored form. Graph the function. (15 points)

$p = \pm 1, \pm 3$

$q = \pm 1, \pm 13$

$\frac{q}{p} = \pm 1, \pm \frac{1}{3}, \pm 13, \pm \frac{13}{3}$

$$3x-1 \overline{\smash{)}\,3x^3 - 13x^2 + 43x - 13}$$

$x^2 - 4x + 13$

$-\ 3x^3 + x^2$

$-12x^2 + 43x$

$+12x^2 \mp 4x$

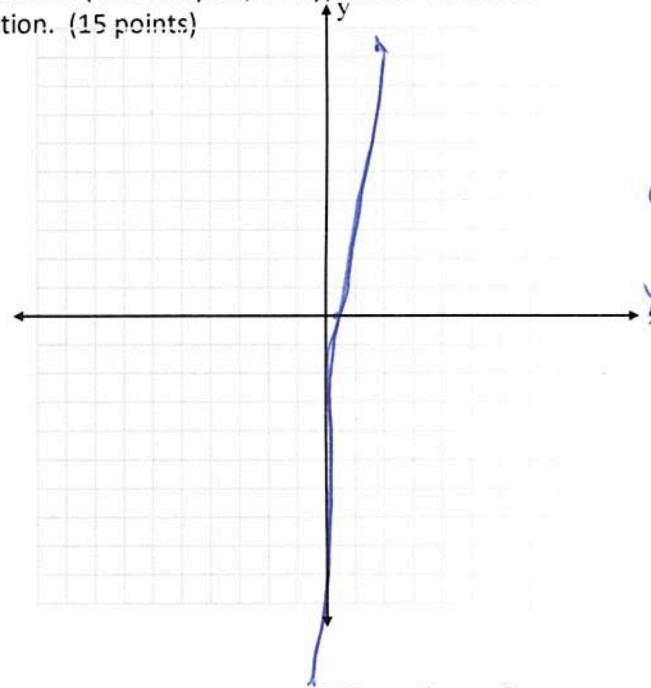
$39x - 13$

$-\ 39x + 13$

0

$(3x-1)(x^2-4x+13)$

$x = \frac{4 \pm \sqrt{16 - 4(1)(13)}}{2(1)}$ complex



one real zero at 1/3

y-int at: -13

4. Use the remainder theorem to find the value of $f(4)$ for the function $f(x) = x^4 - 3x^2 - 13x + 8$ by dividing by $(x-4)$. (10 points)

$$x-4 \overline{\smash{)}\,x^4 - 0x^3 - 3x^2 - 13x + 8}$$

$x^3 + 4x^2 + 13x + 39$

$-\ x^4 + 4x^3$

$4x^3 - 3x^2$

$-\ 4x^3 + 16x^2$

$13x^2 - 13x$

$-\ 13x^2 + 52x$

$39x + 8$

$-\ 39x + 156$

164

$4 \,|\, 1 \quad 0 \quad -3 \quad -13 \quad 8$

$\quad\ \ 4 \quad 16 \quad 52 \quad 156$

$\quad 1 \quad 4 \quad 13 \quad 39 \,|\, 164$

$f(4) = 164$

Part 2: In this section you will record your answers on paper along with your work. After scanning, submit them to a Canvas dropbox as directed. These questions will be graded by hand.

5. Divide using polynomial long division. Write the solution as $Quotient + \frac{Remainder}{Divisor}$. (12 points each)

a. $\frac{4x^5-8x^4+x^3+x^2+x}{2x^3+1}$

$$\boxed{2x^2-4x+\frac{1}{2} \quad + \quad \frac{-x^2+5x-\frac{1}{2}}{2x^3+1}}$$

$$2x^3+1\,\overline{)\,4x^5-8x^4+x^3+x^2+x+0}$$

$$-4x^5 \qquad \mp 2x^2$$

$$-8x^4+x^3-x^2+x+0$$

$$+8x^4 + \qquad 4x$$

$$x^3-x^2+5x+0$$

$$-x^3 \qquad \mp\frac{1}{2}$$

$$-x^2+5x-\frac{1}{2}$$

b. $\frac{x^4+x^3-2}{x+1}$

$$x^3$$

$$x+1\,\overline{)\,x^4+x^3+0x^2+0x-2}$$

$$-x^4 \mp x^3$$

$$-2$$

$$x^3 - \frac{2}{x+1}$$

6. Redo problem 5b: $\frac{x^4+x^3-2}{x+1}$ using Synthetic Division. Show that the answer you obtain is consistent with your answer in 5b. (10 points)

$$\begin{array}{r|rrrrr} -1 & 1 & 1 & 0 & 0 & -2 \\ & & -1 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & \boxed{-2} \end{array}$$

it is consistent

$$x^3 \qquad + \frac{-2}{x+1}$$

7. Write a polynomial with the given properties. You may leave the polynomial in factored form **with real coefficients**. (12 points each)

a. $n = 3$, zeros are $-1, 5$ (multiplicity two) and $f(+1) = 52$

$$a(x+1)(x-5)^2$$

$$f(-1) = 52 = a(2)(-4)^2 = a(32)$$

$$a = \frac{52}{32} = \frac{13}{8}$$

$$\boxed{f(x) = \frac{13}{8}(x+1)(x-5)}$$

b. $n = 4$, zeros $2, -5, 3 - 2i$, and $f(1) = -24$

$$a(x-2)(x+5)(x-3+2i)(x-3-2i)$$

$$x^2 - 3x - \cancel{2i}x - 3x + 9 + \cancel{6i} + \cancel{2i}x - \cancel{6i} - 4\overset{(-1)}{i^2}$$

$$x^2 - 6x + 13$$

$$a(x-2)(x+5)(x^2-6x+13)$$

$$f(1) = -24 = a(-1)(6)(1-6+13) = a(-6)(8) = -48a \quad a = \frac{-24}{-48} = \frac{1}{2}$$

$$\boxed{f(x) = \frac{1}{2}(x-2)(x+5)(x^2-6x+13)}$$

8. Sketch the graph of the function $f(x) = \frac{x^2-1}{x^2-4}$, but finding i) any intercepts, ii) any vertical asymptotes or holes, iii) any horizontal or slant asymptotes. (20 points)

VA = x = 2, x = -2

HA = y = 1

$$\frac{(x-1)(x+1)}{(x-2)(x+2)}$$

int 4