10/24/2024

Exponential Functions (Ch 6 in the online textbook)

Changes to the schedule coming up:

- 11/5 is election day, but I have a conflict with class time on 11/7 so I will record class on 11/5 at the usual time. You are welcome to attend live, however, I will post the recording for you to view on 11/7. I will hold office hours at 8 p.m. for anyone with questions.
- 11/14 is a review day for Exam #3. Since I expect the class to be relatively short, we will start at 8 p.m. instead of 7 p.m.
- The third exam, currently scheduled for Tuesday, 11/19 will be moved to 11/21 and we'll meet on 11/19 to cover the material on linear equations originally scheduled for 11/21.
- Class on 12/5 will be cancelled. We probably won't need two classes to cover the partial fractions topic and if we do need additional time, we can use some of the review lecture on 12/10 to complete whatever remains or do additional examples.

I will post these changes to an announcement as well.

Exponential Functions
Have the form:

$$f(x) = (a^x)$$

a is called the base of the exponential.
It is a constant.

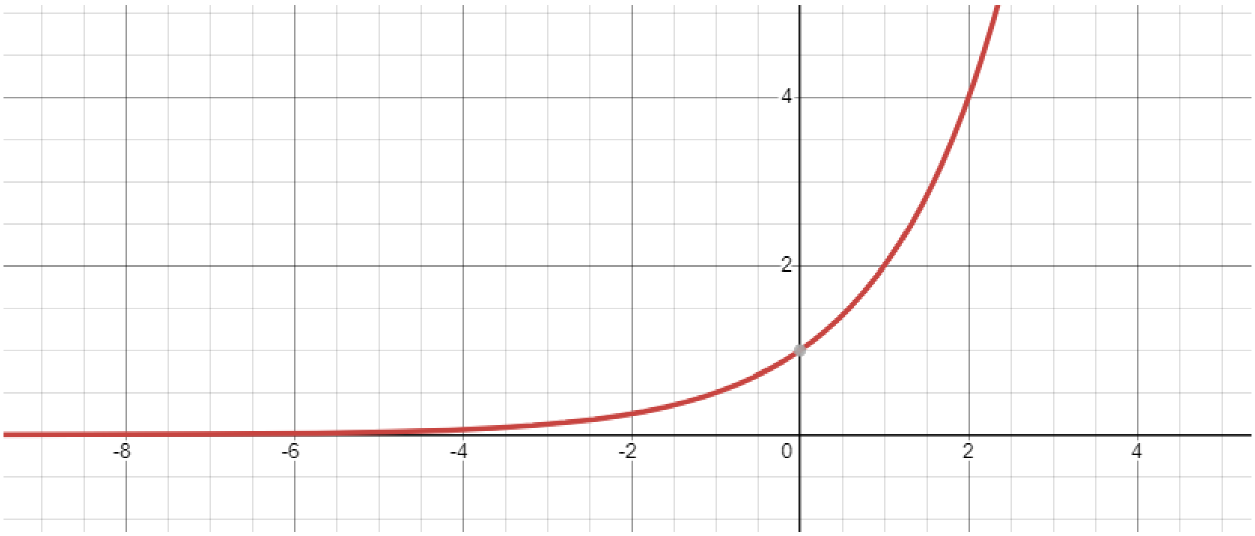
$$0 < a < 1, or\ a > 1$$

The only reason 1 is not allowed is because it's boring: the function is always 1 since 1 to any power is always 1. (similar argument for 0, you can't also do negative exponents for 0)

a does have to be positive to be a smooth, continuous function

Think about a specific example, $f(x) = 2^x$
If we tried to plot this function, we would choose specific values for x and then connect the dots.

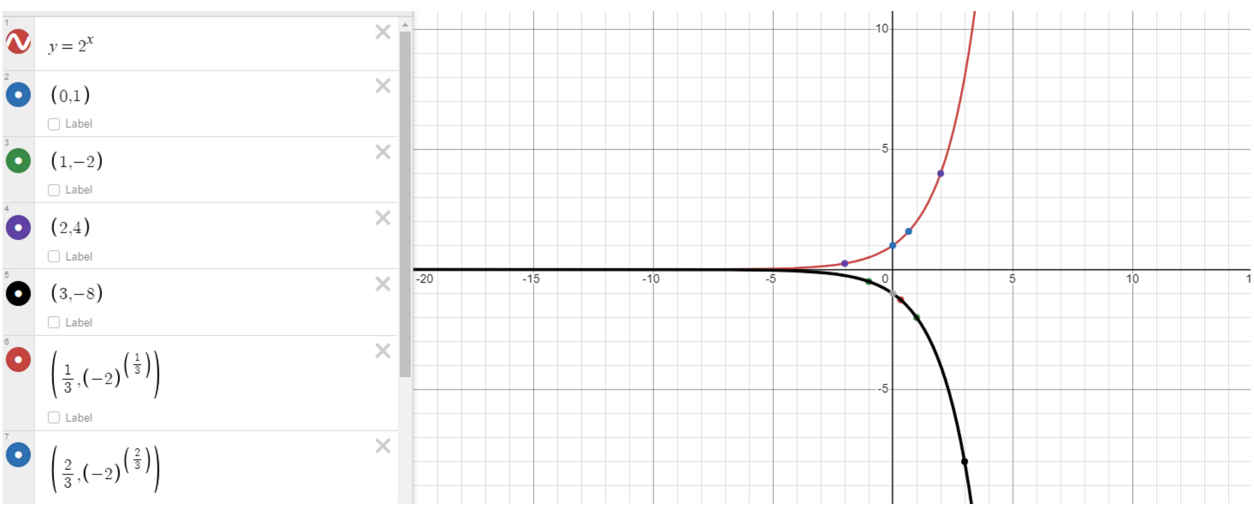
x	$f(x) = 2^x$
0	$f(0) = 2^0 = 1$
1	$f(1) = 2^1 = 2$
2	$f(2) = 2^2 = 4$
$\frac{1}{2}$	$f\left(\frac{1}{2}\right) = 2^{1/2} = \sqrt{2}$
-1	$f(-1) = 2^{-1} = \frac{1}{2}$
-2	$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
$-\frac{1}{2}$	$f\left(-\frac{1}{2}\right) = 2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$



What happens if I use a negative value for the base? Instead of 2, use -2?

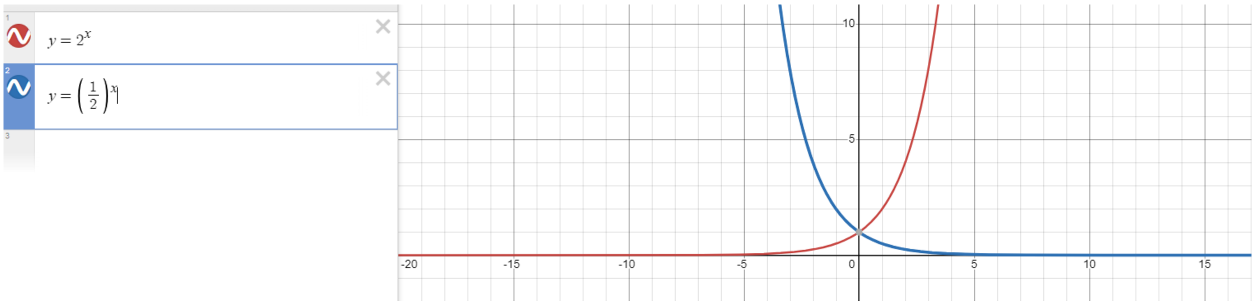
$$g(x) = (-2)^x$$

x	$f(x) = (-2)^x$
0	$f(0) = (-2)^0 = 1$
1	$f(1) = (-2)^1 = -2$
2	$f(2) = (-2)^2 = 4$
$\frac{1}{2}$	$f\left(\frac{1}{2}\right) = (-2)^{1/2} = \sqrt{-2}$
-1	$f(-1) = (-2)^{-1} = -\frac{1}{2}$
-2	$f(-2) = (-2)^{-2} = \frac{1}{2^2} = \frac{1}{4}$
$-\frac{1}{2}$	$f\left(-\frac{1}{2}\right) = (-2)^{-\frac{1}{2}} = \frac{1}{(-2)^{\frac{1}{2}}} = \frac{1}{\sqrt{-2}}$



So, the base must always be positive.

If the base is $a > 1$, the exponential function will increase as x increases, but if the base is $0 < a < 1$, the exponential function will decrease as x increases.



All bases of the exponential function pass through the point (0,1).
Another point on every exponential function is $(1, a), and\ \left(-1, \frac{1}{a}\right)$.

Exponential functions have a domain of all real numbers (assuming the base is positive)
Their range is $(0, \infty)$ (i.e. 0 is not included)

Compare bases $a > 1$

Compare bases $0 < a < 1$

Applying transformations to our functions for graphing.

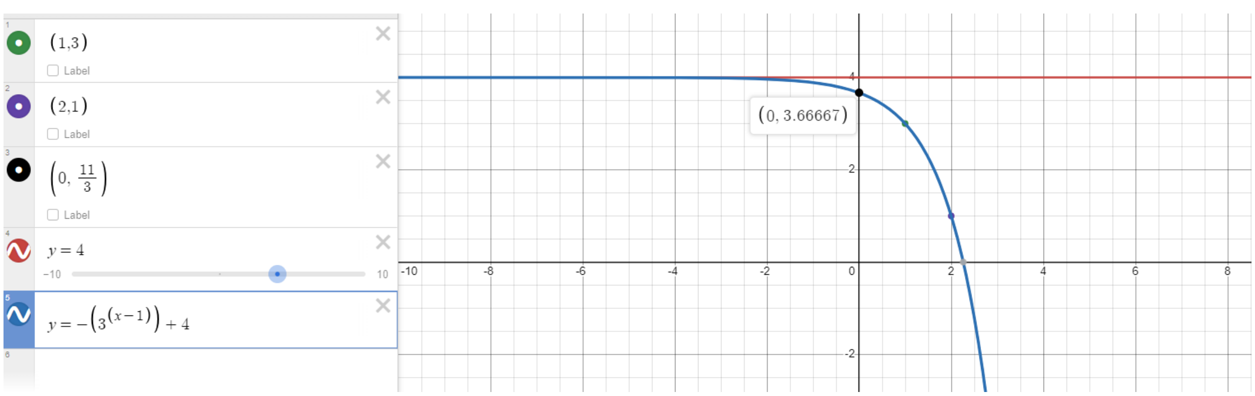
$$f(x) = -(3^{x-1}) + 4$$

The x-1 in the exponent shifts the graph right by 1 (this does not change the domain)
The negative out front is a vertical reflection (the range is changed to $(-\infty, 0)$)
The 4 on the end is a vertical shift up. (the range is changed to $(-\infty, 4)$)

This 4 is now the horizontal asymptote (0 was before, now it's 4)

Key points: from 3^x base function

	$(0,1)$	$(1,3)$	$\left(-1,\frac{1}{3}\right)$
horz shift	$(1,1)$	$(2,3)$	$\left(0,\frac{1}{3}\right)$
v. refl.	$(1,-1)$	$(2,-3)$	$\left(0,-\frac{1}{3}\right)$
v. shift	$(1,3)$	$(2,1)$	$\left(0,\frac{11}{3}\right)$



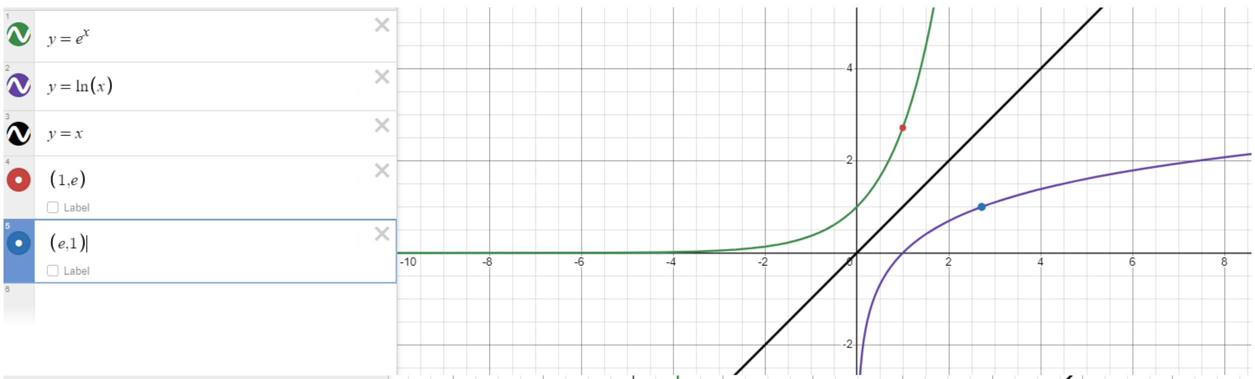
Logarithmic Functions are the inverse of the exponential function

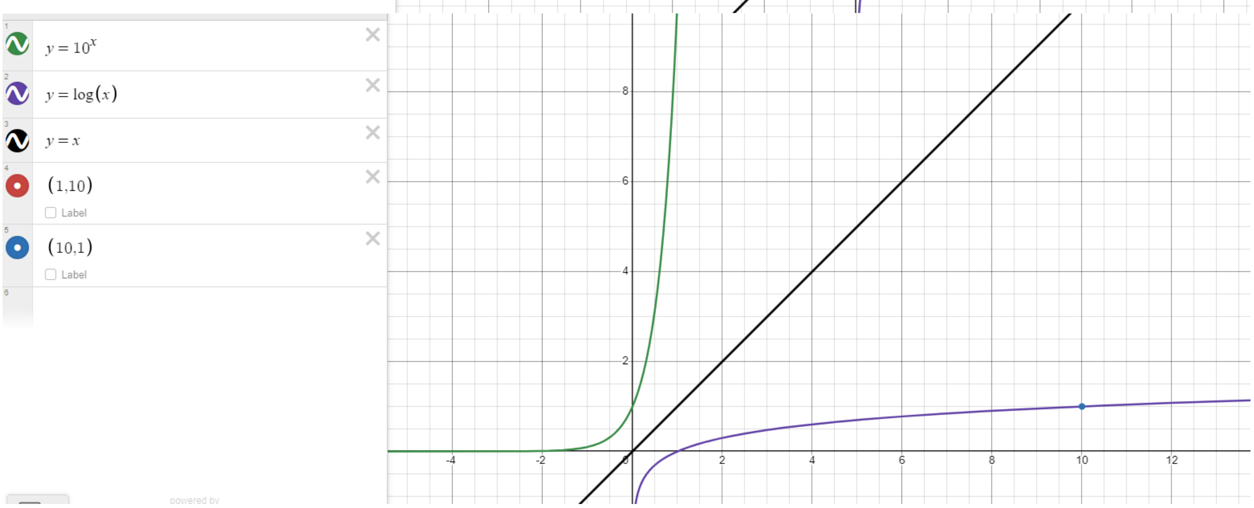
Log functions have bases, just like exponential functions, and they also must be positive (and not 1 or 0).

$$\log_a(y) = x$$
$$a^x = y$$

If $(0,1), (1, a), \left(-1, \frac{1}{a}\right)$ is on the graph of the exponential function $f(x) = a^x$, then the points $(1,0), (a, 1), \left(\frac{1}{a}, -1\right)$ are on the graph of the $f^{-1}(x) = \log_a(x)$.

$\log x$ without a base noted is assumed to be $\log_{10} x$ (base-10)
Natural log is written as $\ln(x) = \log_e(x)$





Logarithms have a domain of $(0, \infty)$
Have and range of all real numbers