10/29/2024

Logarithmic Functions

Last time, we discussed exponential functions. We said that the domain of $f(x) = a^x, a > 0, a \neq 1$ is going to be $(-\infty, \infty)$ i.e. all real numbers, and the range $(0, \infty)$.

Logarithms are the inverse of exponential functions and so the domain of $g(x) = \log_a x\,, a > 0, a \neq 1$ is $(0, \infty)$ and the range is $(-\infty, \infty)$.

Because they are inverses of each other $\log_a(a^x) = x$, $\log_3(3^4) = 4$.
Similarly, $a^{\log_a x} = x$, e.g. $10^{\log 5} = 5, e^{\ln 26} = 26$

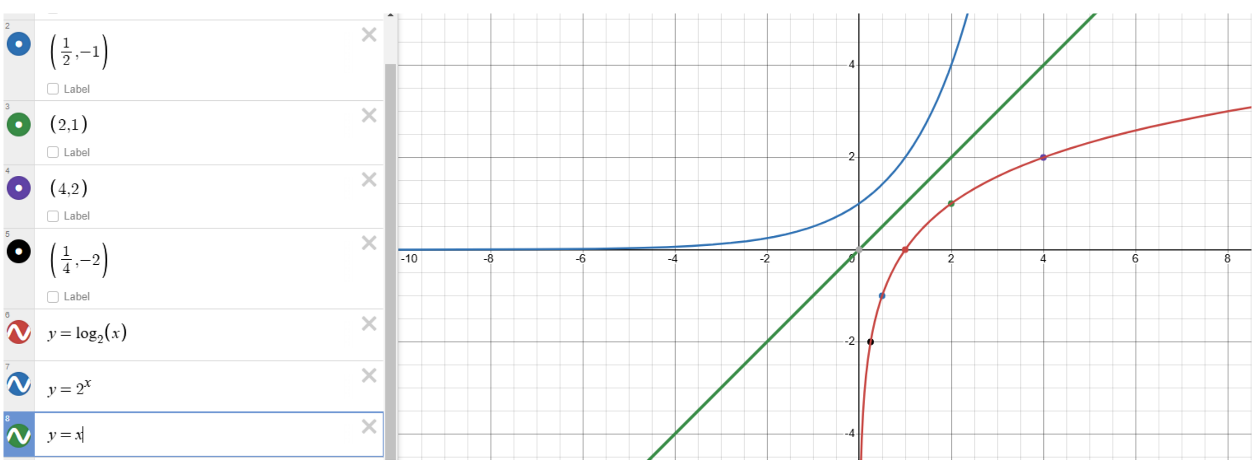
$$\log_3 10 = x$$
$$3^x = 10$$
$$x \approx 2.0959$$
$$3^{2.0959} \approx 9.999964 \ldots$$

On an exponential function $f(x) = a^x$, we have the points $(0,1), (1, a), \left(-1, \frac{1}{a}\right)$.
On any log function we will have $g(x) = \log_a(x)$, will have the points $(1,0), (a, 1), \left(\frac{1}{a}, -1\right)$

$$\log_a(1) = 0$$
$$\log_a(a) = 1$$
$$\log_a\left(\frac{1}{a}\right) = \log_a(a^{-1}) = -1$$

Plot the graph of $g(x) = \log_2(x)$

Key points: $(1,0), (2,1), \left(\frac{1}{2}, -1\right)$



Applying transformations to the key points

$$h(x) = -2\log_2(x-1) + 4$$

Horizontal shift right by 1
Vertical reflection (negative sign out front)
Vertical stretch by factor of 2
Vertical shift up by 4

Keypoints from $\log_2(x)$	$(1,0)$	$(2,1)$	$\left(\frac{1}{2}, -1\right)$
Horizontal shift right 1	$(2,0)$	$(3,1)$	$\left(\frac{3}{2}, -1\right)$
Vertical reflection	$(2,0)$	$(3,-1)$	$\left(\frac{3}{2}, 1\right)$
Vertical stretch by 2	$(2,0)$	$(3,-2)$	$\left(\frac{3}{2}, 2\right)$
Vertical shift up 4	$(2,4)$	$(3,2)$	$\left(\frac{3}{2}, 6\right)$

The domain is now $(1, \infty)$ because of the rightward shift by 1,

$$(0, \infty) + 1 \rightarrow (1, \infty)$$

Domain of the log is $x > 0$

$$f(x) = \log(g(x))$$
$$domain: g(x) > 0$$

$$f(x) = \ln\left(\frac{x-2}{x}\right)$$

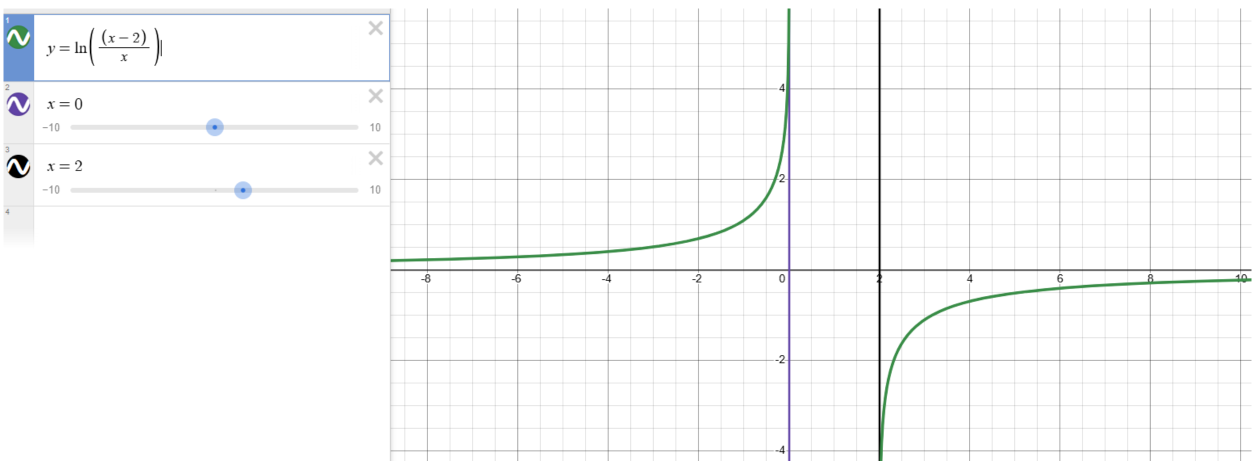
What is the domain of this function?

$$\frac{x-2}{x} > 0$$

Set both the denominator and the numerator equal to 0, these are the places where the sign changes, and then we build a sign chart.

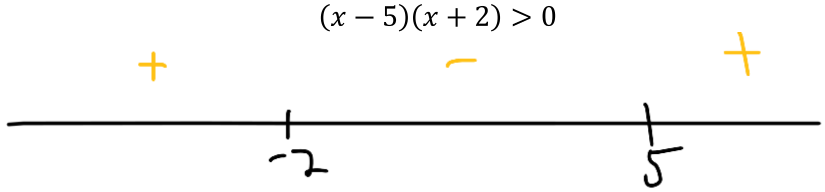
$$x - 2 = 0, x = 2$$
$$x = 0$$

Domain: $(-\infty, 0) \cup (2, \infty)$



$$f(x) = \log(x^2 - 3x - 10)$$

$$x^2 - 3x - 10 > 0$$
$$(x-5)(x+2) > 0$$



Domain: $(-\infty, -2) \cup (5, \infty)$

Change of base formulas

$$\log_a b = \frac{\ln b}{\ln a} = \frac{\log b}{\log a}$$

$$\log_3 10 = \frac{\ln(10)}{\ln(3)} \approx 2.095903274 \ldots.$$

$$f(x) = \log_2(x) = \frac{\ln(x)}{\ln(2)}$$

Occasionally, you'll encounter problems with the variable in the base of the log….

$$\log_x(4) = 3$$

Convert this into an "exponential" version of this expression:

$$x^3 = 4$$
$$x = \sqrt[3]{4}$$

Next time, we'll talk about properties of logs (and exponentials)